Zero Intelligence in an Edgeworth Box

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Abstract

Gode and Sunder (1993) brought a lower-bound to the literature on bounded rationality in the continuous double auction, asserting the institutional rules, rather than the behavior of traders, provide the equilibrating tendencies seen so often in the laboratory. Gode et al. (2004) brought these 'zero intelligence' traders to the general equilibrium paradigm as well, though not before a group of studies began questioning whether this new lower-bound was truly void of intelligence. This paper tests the driving assumptions of the general equilibrium adaptation of the zero intelligence model. I find significant variation in market performance when adjusting enforcement of five different assumptions. Enforcement of behavioral-oriented and market-oriented rules show stark differences in their influence on market outcomes, with behavioral-oriented rules providing the most guidance. [1]

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[1]Many thanks to Daniel Friedman for guidance and support, as well as comments and advice from Jacob Goeree, Bernardo Garcia-Pola, Natalia Lazzati, Kristian López Vargas, and members of the UCSC experimental and theoretical workshop BETE. I also greatly thank an anonymous associate editor and two anonymous referees for their helpful comments and feedback. Any errors are my own.
Declaration of Interest: None
Funding: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Keywords: Continuous Double Auction, General Equilibrium, Exchange Economy, Zero Intelligence

JEL Classifications: C63, D44, D47, D51, D80

"[T]he main stream for the last 50 years has entered the wilderness of bounded rationality through the rationality gate... but there have been a few who have entered through the other (zero intelligence) gate... both of these groups are needed... though we need 'tunnelers' from both directions and we don't have enough coming from the minimal intelligence direction."
-J. Doyne Farmer (2020) [2]

1 Introduction

A growing literature on boundedly rational behavior in economic agents has been flourishing, and is long-lived in the study of markets (Conlisk, 1996). In particular, agent-based models have been a prominent process by which researchers have entered the zero/minimal intelligence gate of the 'wilderness of bounded rationality' (Axtell and Farmer, 2022). The study of trader behavior and price formation in markets, including the continuous double auction (CDA) institution, has especially benefited from such simple, parsimonious modelling given how easily a market's underpinnings can become over-complicated (Farmer, 2003).

Gode and Sunder (1993) postulated a model for double auction behavior which enters through the gate of no intelligence in a quite literal sense, introducing "zero intelligence" traders to the literature.[3] Zero intelligence traders provide a counter to the traditional rational traders who hold perfect utility maximizing capabilities when placing orders. Instead, prices are randomly chosen within a set of allowable prices ranging from 0 to some maximal price M. If these traders are 'unconstrained', then this price rule is the sole rule determining their trading behavior outside of the rules of the market institution itself. However, if instead 'constrained', the ZI traders (now called ZI-C) follow a no-loss rule, or as erroneously referred to in Gode and Sunder (1993) a "budget constraint." This no-loss rule updates the

[2]This quote is from The First Conference on ZI/MI Intelligence Agents in 2020, in a discussion on agent-based modelling in economics.

[3]Hurwicz et al. (1975) was likely the first proposal of minimal or zero intelligence in a pure exchange setting.

distribution which buying and selling agents can draw from, with buyers being capped above by their unit's redemption value and sellers being capped below by their unit cost. A key result of adding the no-loss constraint is that simulated levels of allocative efficiency are much closer to those of laboratory markets.

Cliff and Bruten (1997)[4] called attention to this claim, providing mathematical and simulated evidence refuting the idea that 'zero intelligence is enough' to provide equilibrium results. The authors claim that since the supply and demand schedules endowed to the ZI traders in Gode and Sunder (1993) create a distribution of potential trade prices that is single peaked at the equilibrium price, constrained ZI agents trading near the CE price is unsurprising. Cliff and Bruten simulate two economies with schedules that create 'box' trade distributions, eliminating the guidance of the single-peakedness of the original example. Both example economies confirm Cliff and Bruten's suspicions, with allocative efficiency falling drastically for the ZI-C traders. The other major complaint against the ZI-C traders was made in Gjerstad and Shachat (2021).[5] Rather than questioning the original example parameters, Gjerstad and Shachat question the intelligence imparted by the no-loss constraint. The authors show such a constraint is equivalent to assuming individual rationality, which is certainly more than zero intelligence. Other papers also contributed to the questioning of zero intelligence agents soon after, either by analyzing specific aspects of the model or presenting adjustments or refinements in new models (e.g. Cliff and Bruten (1998)).

Following the push-back, the authors pushed zero intelligence into more complex market settings. First, Gode and Sunder (2004) studied ZI traders in partial equilibrium when price restrictions are present, arguing ZI trader behavior is immune to such rules.[6] Second, Gode et al. (2004) brought ZI traders to a more complex setting in general equilibrium;

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[4]This paper is one of a group of working papers or technical reports by the authors on such a topic.

[5]Though the two papers seem far apart in age, Gjerstad and Shachat's retort against ZI-C traders has been around since 1996.

[6]Though, the authors do acknowledge to some degree the push-back when stating "ZI traders avoid losses" and not mentioning a budget constraint.

given the price independence story of partial equilibrium doesn't always capture real-world markets, such a model had noticeably been missing. Given the state of experimental interface technology at the time, a simulation-based test held a considerable advantage over experimental tests in the busier general equilibrium setting. This new ZI model (ZI-GE henceforth) translated the heart of ZI traders behavior in two steps: (1) traders place orders via a uniform price draw and a quantity restriction, and (2) traders avoid losses by placing only utility-improving orders.

Both the partial equilibrium (ZI-C) and general equilibrium versions of the ZI model rely on a few restrictive assumptions, either institutional or behavioral in nature. In the spirit of Cliff and Bruten (1997) and Gjerstad and Shachat (2021), this paper provides a thorough examination of the robustness of zero intelligence traders in GE (as in Gode et al. (2004)). Using simulations, I test crucial assumptions made in defining the behavior of ZI-GE traders and the design of the continuous double auction they trade in. As with the partial equilibrium critiques, these simulations support the "zero intelligence is not enough" sentiment, in that the behavioral assumptions of GSS provide a substantial improvement in ability and thus market efficiency.

Five assumptions made in Gode et al. (2004) (henceforth GSS) are considered in the simulation exercise, many of which have been present in prior works on the continuous double auction. A spread-reduction rule, which forces new orders to shrink the current best bid-ask spread, an orderbook reset policy, which resets the orderbook after every trade, and a single unit quantity restriction form the set of market rules considered. The behavioral assumptions tested are the no-loss constraint imposed in both Gode and Sunder (1993) and Gode et al. (2004) and the price decision process made in Gode et al. (2004). By testing all 32 combinations of these five assumptions being enforced (or not), I find significant variation in key market performance outcomes. Allocative efficiency ranges from 0.34 to 0.94, while average price deviations are as high as 13.06 and as low as 0.46 for a parametrization with a competitive equilibrium price of 2.44. Measures of price volatility and trade volume and flow are also largely impacted by changes in the assumptions.[7] Behavioral-oriented constraints appear to provide the majority of the variation in outcomes, implying, much like Cliff and Bruten (1997), that 'zero' is indeed not enough. In other words, trader behavior seems to provide some of the basis for market equilibration, and the institution itself is not solely responsible.

Along with the zero intelligence literature, this paper belongs to the more general literature on agent-based modelling in markets. A stream of other agent-based models emerged in the late 20th century alongside Gode and Sunder (1993). Wilson's (1987) game theoretic venture, while informative, showed the limitations complexity places on a strategic approach to modelling CDA trader behavior. Friedman (1991) and Easley and Ledyard (1993) both posited non-strategic models with variations on a reservation price mechanism, yet differed in the dynamics of interest. Friedman studied within-period pricing dynamics, while Easley and Ledyard focused on across-period dynamics. Gjerstad and Dickhaut (1998) proposed a belief-based model shortly after, though moving away from zero intelligence towards higher complexities of trader intelligence. Each model in this group is oriented in a partial equilibrium setting; very few models have focused on general equilibrium dynamics in these simple CDA markets (Gode et al. (2004) and Crockett et al. (2008) are a couple such papers). [8]

The rest of the paper continues as follows. Section 2 recounts the zero intelligence models of Gode and Sunder (1993) and Gode et al. (2004), and provides adjustments to the key assumptions in the models. Section 3 maps out a vast simulation test of the underlying determinants of the model and environment, while Section 4 analyzes the full factorial of market configurations and provides insight in potential applications of the model and simulations to experiments. Section 5 concludes the paper.

[7] Robustness runs confirm similar results in multiple versions of constant elasticity of substitution preferences (including a Cobb-Douglas parametrization from Gode et al. (2004) and CES preferences with strong income effects (an exponent of -1) from Gjerstad (2013).

[8] See Axtell and Farmer (2022) for a recent survey on the agent-based modelling literature in economics.

2 Zero Intelligence

The continuous double auction (CDA) is a ubiquitous market institution, used in markets around the world. Traders place orders to either buy or sell some units, often one, of a good. An order contains three key pieces of information: a price, a quantity and a time of placement. The 'continuous' part of the continuous double auction comes from the third piece, as traders may place orders in continuous time throughout the duration of the market or trading period. These orders are arranged in an orderbook, separated based on whether the request is to buy or sell the good. Orders are ranked by price and then by time, with the buy orders being ranked from highest to lowest price and sell orders ordered from lowest to highest price. Orders of the same type and price are then ranked by time of placement from earliest to latest. Trades occur when two orders 'cross', meaning the price of a buy order is higher than the price of a sell order. The trade occurs at the price of the trading order that appeared first in the orderbook, i.e. the 'crossed' order (as opposed to the 'crossing' order). Within a given market period or realization, multiple prices across trades may occur.

Gode and Sunder (1993) present a model of trader behavior in the CDA, with the focal point being the lack of intelligence imparted on the traders. The model uses a version of the CDA commonly used in market CDA experiments, as characterized below. Like many other works in the continuous double auction literature, both theoretical and experimental, the economy is defined by the trade of a single good by two types of traders: buyers and sellers. Orders are restricted to being one unit, and each of the n traders may only have one order in the orderbook at a time. The model considers a partial equilibrium framework, whereby prices in the market for the single good in focus don't impact the markets for any other good in these traders' universe. As such, trader preferences are simplified to demand and supply schedules. Each buyer has a demand schedule of redemption values for the m units of the good they desire and each seller similarly has a supply schedule of unit costs for each of the units they produce.[9]

Traders enter the market uniformly randomly, one at a time, to place an order. Here, 'enter' means interacting with the market with the intent to place an order. The side of the orderbook they place the order in is referred to here as the 'side of entry'. The trader constructs their order by uniform randomly drawing a price to pair with their single unit desired quantity. In the base, or 'unconstrained', version of the model, the support of prices from which both buyers and sellers draw is $[0, M]$, where M is some maximum allowable price. In the 'constrained' version of the model, ZI-C buyers draw from $U[0, v_i]$ where v_i is the buyer's redemption value for their i^{th} desired unit. ZI-C sellers draw from $U[c_j, M]$, bounded below by the unit cost of their j^{th} unit. The traders all have ordering preferences over their demand or supply schedules when selecting the unit to fill the one unit quantity in their orders. Buyers rank the redemption values highest to lowest, while sellers rank the unit costs lowest to highest. In addition to the single-unit quantity constraint and schedule ordering preference, another key rule was enforced in the model of Gode and Sunder (1993), namely the orderbook is cleared of all orders after the completion of a trade. [10]

2.1 General Equilibrium

Gode et al. (2004) expand on the original zero intelligence model by removing the simplifying assumptions of partial equilibrium and considering a general equilibrium framework instead. The setting thus expands, though only slightly, to a setting with two goods, X and Y. With two goods, two types of traders, and no production, the environment becomes an exchange

[9]Production is assumed to be upon sale, so sellers only incur the unit costs on units of the good they successfully sell.

[10]Though not implemented in their GE-adjusted model (Gode et al. (2004)), or in the markets simulated in this paper, another interesting rule (which, like an orderbook reset, slows the pace of the market) used in Gode and Sunder (1993) is that the two trading agents do not enter the market again until all other traders have traded the same number of units, i.e. if a buyer and seller trade their first units, then every other trader must trade their own first units before the first two may submit orders again. This is not natural in markets and could be harmful to trade and efficiency (as trade and price discovery may be slowed).

economy in an Edgeworth box.

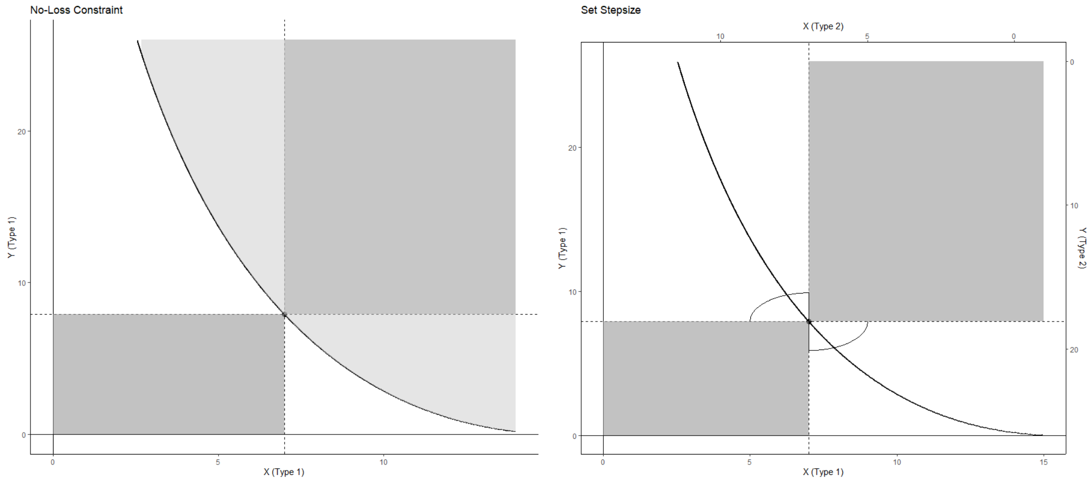

Figure 1: **Left:** No-loss constraint in ZI-GE. Light grey areas are utility improving and feasible reallocations, while dark grey areas are not feasible as they constitute either gains in both goods or losses in both goods. **Right:** Set step-size. The circle (shown as an ellipse due to relative good scaling) centered at the trader's current allocation represents all allocations satisfying a step-size constraint with $r = 2$.

This increase in economy complexity comes with an adjustment in a few of the assumptions made in Gode and Sunder (1993). First, traders no longer hold demand and supply schedules, but instead are assumed to have utility functions defining their preferences over bundles of units, (x, y), of goods X and Y. As such, the no-loss constraint is adjusted to prevent utility loss, instead of profit loss. Figure 1 (left) visualizes this constraint's impact on feasible reallocations. Second, order quantities are limited by a step size as opposed to a unit limit. Instead of allowing bids or asks of only one unit, trades must satisfy a vector length constraint which states the distance from the trader's current allocation to the allocation they wish to move to (upon acceptance of their new order) must be equal to a set length. In other words, the step size for all orders is some constant r where $r \equiv \sqrt{(\Delta x)^2 + (\Delta y)^2}$ and Δx and Δy represent the intended adjustment in x and y. As shown in Figure 1 (right), this creates an arc of feasible orders in the bid and ask quadrants (relative to the trader's current endowment) of the Edgeworth box.

The final adjustment to bring their model to general equilibrium is in the price decision.

The price of an order is drawn from a uniform distribution over radians. Without a no-loss constraint, this would mean drawing a radian value (called θ) uniformly in the region weakly 'north-west' of the trader's current allocation (between $\pi/2$ and π holding the current allocation as the origin) when selling X or weakly 'south-east' when buying (between $3\pi/2$ and 2π). However, with the no-loss constraint enforced, this distribution is adjusted to be bounded either above or below by the MRS of the trader (in radians) at their current allocation (call this $\theta(MRS_c)$) depending on whether the trader is drawing for a buy or sell order.[11] Figure 2 shows this price selection process paired with the above two constraints. Combining angle-based price choice with the step-size reduces the support of the distributions even further, as any positive step-size would increase the lowest allowable sell price and highest allowable bid price.

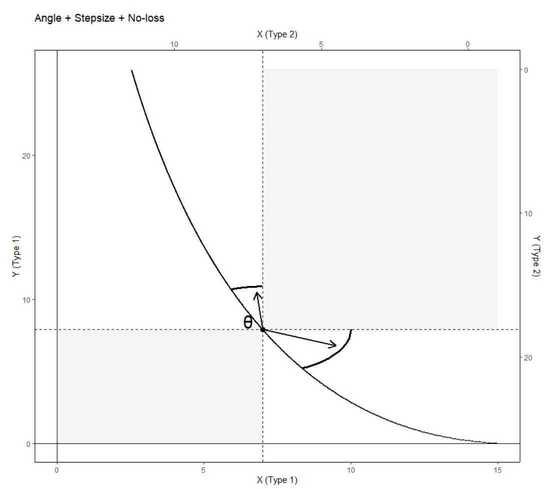

Figure 2: Feasible orders in market with angle-based price choice, no-loss constraint, and a set step-size, as in Gode et al. (2004).

All of the above adjustments in prior assumptions are paired with a loosening of trader roles to define the market. Namely, traders no longer act as only buyers or sellers, but place an order on both sides of the market every time they are drawn.

[11]This then adjusts the distributions to be $U[\pi/2, \theta(MRS_c)]$ for buys and $U[\theta(MRS_c), 2\pi]$ for sells.

2.2 Assumptions and Adjustments

Much like the response to the ZI traders of Gode and Sunder (1993), one could question how minimal, or close to 'zero', intelligence these traders are? Is zero intelligence enough or are the assumptions at play providing the markets price and allocation adjustment with proverbial guide-rails? To this end, a few further adjustments to the assumptions of Gode et al. (2004) can be considered.

First, and most similar to prior critiques of Gode and Sunder (1993), the no-utility-loss constraint can be removed. Analogous, though much more obvious, to the appeal made in Gjerstad and Shachat (2021), such a constraint is equivalent to individual rationality. This would mean traders draw from the base distribution described in Gode et al. (2004). Second, the quantity constraint to create a reallocation bound (via a set step-size) can be loosened, with a natural (and historical) middle-ground being a unit restriction on the commodity (X).

Third, the order choice process can be simpler, at least in execution. Rather than choosing a radian draw, converting this draw to a price, then finding the appropriate x quantity to satisfy the set step-size constraint, the trader could just uniformly draw a desired reallocation from the set of feasible allocations (i.e. (x, y) pairs that are neither better in both goods, nor worse in both). From this desired reallocation, the trader can back out the required price and quantity of the order needed to reach the new bundle (were it to be fully filled). Orders to buy or sell are defined in the following spaces:

$$Buy: \quad [X_{current,i},\ \frac{n}{2} \cdot (X_{endow,b} + X_{endow,s})] \times [0, Y_{current,i}] \tag{1}$$

$$Sell: \quad [0, X_{current,i}] \times [Y_{current,i},\ \frac{n}{2} \cdot (Y_{endow,b} + Y_{endow,s})] \tag{2}$$

$X_{current,i}$ is the X holding of trader i in his current allocation, and $X_{endow,b} + X_{endow,s}$ denotes the total X holding of a buyer-seller pair at the inception of a market. Here n denotes the number of traders in the market. The feasible choice set, under no other assumptions previously discussed, is shown in Figure 3.[12] Though the difference in complexity between the two choice methods is apparent, the impact on potential order placement is considered further in Appendix C.

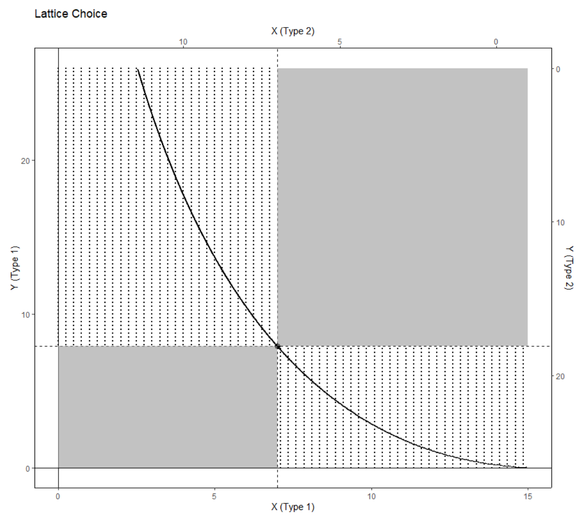

Figure 3: Feasible orders under simpler reallocation/order choice. Here x and y are shown with minimal ticks/increments, creating a lattice. With no minimal tick, the entire second and fourth quadrant relative to the current allocation are feasible orders.

3 A Test of Zero Intelligence

I present a test of the ZI-GE model via a full factorial simulation exercise. Each assumption or rule housed in the model, market, and/or setting is incrementally varied, yielding market outcomes for each potential combination.

[12]Note this visualization considers the case where units of x and y have a minimal tick, or increment, which is often seen in real-world markets. The result is a lattice over which the trader can uniformly draw a desired reallocation (and resulting order to place to feasibly get there).

3.1 Trading Institution

Consistent with the institution of choice in both GS and GSS, the simulations employ the continuous double auction (CDA) trading institution, though in a highly stylized variation. A simple market over two goods, X and Y, provides the space for trade, with Y acting as a numeraire (or a good whose price is normalized to 1). As such, trade can occur strictly using the two goods while using units of Y as a price for commodity X.

As defined in Section 2, this implies a specific structure over the order placement process and order structure. In this two good pure exchange setting, this order structure simplifies to specifying a desired quantity (in units of X), a price (in units of Y per unit of X), and a determination of whether the order is to buy or sell said quantity. The quantity may vary in number of units and contain partial units, implying units of the commodity X are divisible. The price may vary in a similar fashion. While minimal 'ticks', or subunits, are often specified in real-world markets, the more theoretical and computational nature of the simulation exercise does not require such restriction. In effect, the quantity and price variables are continuous, each on $\mathbb{R}_+$, though with upper bounds determined by the holdings of the traders and the total units defining the market as a whole. All orders are contained in the orderbook.

These orders are placed by traders in 'continuous' time, where time is defined by a set number of time increments, or trader entries. In each entry, a uniformly randomly drawn trader places an order to the orderbook, which contains all orders until they are either canceled or filled. An order is filled if the entirety of its desired quantity is traded.

3.2 Environment and Parametrization

Each market contains eight computerized traders programmed to follow the behavior as prescribed in the ZI-GE model. Traders are split evenly into two types, natural buyers and natural sellers. Natural buyers (sellers) have marginal rates of substitution above (below) the competitive equilibrium price; traders are two-way traders as in ZI-GE, however they only place one order at a time to simplify the market. Traders within a type are replicas of each other, meaning they have the same starting endowments and preferences as each other.

Traders are assumed to have constant elasticity of substitution (CES) preferences over the two goods, X and Y:

$$u(x_i, y_i) = c_\tau((a_\tau x_i)^{\rho_\tau} + (b_\tau y_i)^{\rho_\tau})^{\frac{1}{\rho_\tau}} \tag{3}$$

where subscript τ refers to the type of trader. In general, adjustments in ρ can represent other popular preferences as well, such as Cobb Douglas ($\rho \to 0$), Leontief ($\rho \to -\infty$) or perfect substitutes ($\rho = 1$). CES preferences are chosen as prior experimental literature (Williams et al. (2000) and Gjerstad (2013)) has induced subjects with such preferences. Additionally, given Gode et al. (2004) uses relatively standard Cobb-Douglas preferences, a ρ somewhere in $(-1, 1)$ can maintain a similar testbed while being close to previous experimental works. The parameters used in this paper's simulations are as shown in Table 1. The (a, b) values follow the convention from Williams et al. (2000), in that they sum to 1; the relative ratios in the a and b values for buyer and sellers are meant to induce a higher preference for the non-numeraire in the buyers than the sellers. The endowments were chosen close to a corner of the Edgeworth box to allow for a decently large lens and contract curve, and thus room for more movement in allocation (and possible prices) throughout the market.

	c	a	b	ρ	(x_{Endow}, y_{Endow})
Buyers	0.113	0.825	0.175	0.5	(3,23)
Sellers	0.099	0.6875	0.3125	0.5	(11,3)

Table 1: Simulated Agent Parameters.

A subset of the simulation process is also run using a parametrization from Gode, Sunder and Spear as well (see Appendix H.2).

3.3 Equilibrium

The environment defined above determines a unique competitive equilibrium price, which implies equilibrium final allocations for all traders. To determine said price, the excess demand for a good, consider X as it is non-numeraire, is solved for a root, or the price which zeroes out the market's excess demand. Each trader i possesses some excess demand, Z_i over good X, defined as difference in their inverse demand for X and their starting endowment of X. For the CES preferences described above, this is written as

$$Z_i^X(p|(x_{i,o}, y_{i,o})) = \frac{a^{\gamma_i}(y_{i,o} + px_{i,o})}{p(a^{\gamma_i} + p^{\gamma_i} b_i^{\gamma_i})} - x_{i,o} \tag{4}$$

where where $(x_{i,o}, y_{i,o})$ is the initial bundle of trader i and $\gamma_i = \frac{r_i}{1-r_i}$.

The individual excess demands are aggregated to find the market's excess demand for X, with the competitive equilibrium price solving

$$Z^X \equiv \sum_i Z_i^X = 0 \tag{5}$$

For the parametrization given in Table 1, the equilibrium price is 2.44. This is true for any realization of this market so long as the number of natural buyers equals the number of natural sellers. By entering the CE price into the buyer's excess demand function, it is revealed that the buyer desires roughly 5.2 units of X in equilibrium, at the CE price. Accordingly, the equilibrium final allocation for natural buyers in this market is $(8.2,\ 10.31)$. A similar process gives a final natural seller allocation $(5.8,\ 15.69)$.

The exact assumptions and market rules are determined at the simulation level, while endowment determination is at the period level and orders/trades occur at the entry level. The set of constraints, as well as the simulation design and technical notes, are described in the next subsection.

3.4 Design

A set of five constraints define the 'treatment' factors in this simulation test: (1) spread reduction rule , (2) single-unit quantities, (3) angle-based price choice process, (4) orderbook resetting, and (5) no-loss constraint. Each factor has two levels, either 0 or 1, representing exclusion or inclusion in the simulated market; see Table 2 for a summary of the definitions given in Section 2. These constraints can be partitioned into two types, behavioral assumptions and market rules. Constraints (3)and (5) are crucial behavioral assumptions made in ZI-GE, while (1),(2) and (4) are market rules enforced in the CDA of ZI and/or ZI-GE.

Given the distinction between the two rule sets, expectations over the implications of each in isolation may differ. As the names may suggest, behavioral assumptions are those that impart some intelligence (such as individual rationality) or more sophisticated order choice process or capability. Market rules, on the other hand, either adjust the allowable structure of orders (as in a single unit restriction), or adjust the criterion for which orders may either enter or exist in the orderbook (such as the case when imposing a spread reduction rule or and orderbook reset rule, respectively).

A few technical notes on the implementation and design choices made with respect to these constraints should be made. While the angle-based price-choice process has been defined fully under the enforcement of a set step-size, the choice of quantity is currently undefined when such a rule is not enforced. This is important as such a scenario happens in all simulated markets with no quantity restriction and with an angle-based process, i.e. one in four markets in this simulation. Figure 4 displays a reasonable process for such a case. Once an angle (θ) has been drawn, the quantity (or units of x) is uniformly randomly drawn along the price vector at the drawn angle. If the no-loss constraint is not enforced, then the upper bound on the quantity choice is determined by the bounds of the Edgeworth box. However under a no-loss constraint, the rule for the upper bound is piece-wise and depends on the drawn angle. If the entirety of the price vector is utility improving, the upper bound

Definitions

Rules:	Orientation	Description: Rule Not Enforced	Description: Rule Enforced
Spread Reduction (SR)	Market	Any order is posted to the orderbook	New orders must improve the best bid-ask spread; new buys (sells) only post to the orderbook if they are higher-priced (lower-priced) than the best buy (sell)
Single Unit (SU)	Market	Order quantities are divisible and can be multiple units	Orders are restricted to have a quantity request of 1 (unit of X)
Lattice/Angle (LA)	Behavioral	Price and quantity derived from uniform random draw over allocations in feasible space	Prices are drawn via a uniform distribution over radians
Orderbook Reset (OBR)	Market	Non-trading orders remain in the orderbook after a trade occurs	After a trade occurs, the state of the orderbook will be reset, with all non-trading orders being removed
No Loss (NL)	Behavioral	Orders may be utility-reducing	Orders may only be placed if they weakly improve the trader's utility upon fully filling

Table 2: Definitions and notation for design rules.

is determined again by the Edgeworth box, but if the price vector is not entirely improving, then the upper bound is the x associated with the intersection of the price vector and the indifference curve.

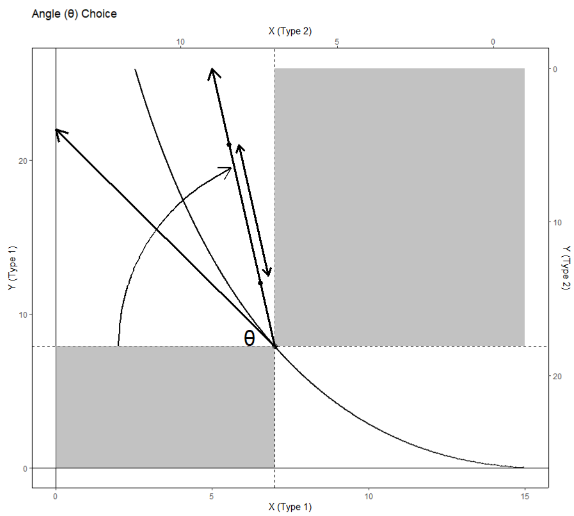

Figure 4: Quantity decision for angle-based order choice when a set step-size is not enforced.

The second technical note is less of a market design choice and more of an experimental design choice. With respect to the set step-size constraint, the determination or selection of a specific r to test is arbitrary. As shown in Appendix B, this selection has large bearings on the performance of the simulated markets and is quite sensitive. Given how arbitrary the selection of r is and how sensitive the market response is to it, I test a milder, or more appropriately, older, quantity constraint. Namely, I include a single-unit restriction as the simulated quantity constraint. Such a selection seems appropriate for a few reasons: (1) the vast majority of the partial equilibrium literature assumes such a constraint, (2) the constraint is not uncommon in the general equilibrium experimental literature (e.g., Gjerstad (2013)), and (3) Gode et al. (2004) reference such a constraint in relation to the set step-size assumption. With respect to (3), Gode et al. claim such a choice has asymmetric impacts on the two goods in the market. I consider this in the analysis, however, restricting only x to a single-unit constraint and allowing y to move freely (i.e. as a numeraire or cash) mimics many real-world settings with indivisible units and wide price dispersion, so it seems like a relevant dimension to include.

Finally, the lattice reallocation choice rule is run taking the x and y axes as continuous, i.e. making the lattice ticks infinitely small.[13] As mentioned in Section 2.2 and Figure 3, this just simplifies the choice process to drawing a point from a continuous rectangle (see sets 1 and 2), where any point is a feasible reallocation.

Wholly, these factors combine to create a full factorial (simulated) experimental design with a total of 2^5 treatments. The five main effects and 27 interactions are tested in Section 4, with the simplest, or lowest level of 'zero', being the control/holdout. Each of the 'factors' in the design are named for analysis as follows: spread reduction (SR), single unit (SU), lattice/angle (LA), orderbook reset (OBR), and no loss (NL). For each of the 32 variations, I run 250 simulations. Each simulation has 3600 market entries across 12 rounds, i.e. 300 entries per round (market realization).

3.5 Outcomes

To analyze the respective performances of full factorial of market orientations, a set of outcomes in price, allocation, market participation, efficiency and convergence are all examined.

In price space, outcomes can either reflect the approximate slope from starting to final allocation or the variation across trades between these points. Average price and per-unit average price are reported to describe the former; average price gives all trades equal weight (i.e. disregards quantity), while per-unit average price weights each trade by its quantity (alternatively, this can be described as dividing the total units of Y traded by the total units of X traded in the round). To capture price variation or volatility, again two metrics are used: namely average absolute price deviation from the competitive equilibrium price, and root-mean-squared error. Root-mean-squared error, or RMSE, is defined as the square root

[13] In practice, this continuity is limited by computer precision, which in R is 1e-16. The command runif() was used to draw values, drawing from a continuous support.

of the average of squared deviations from CE price, or

$$RMSE \equiv \sqrt{\frac{1}{n}\sum_{i}^{n}(P_i - P_{CE})^2}$$

In allocation space, measures of marginal rates of substitution reflect how effective traders have been in reallocating throughout the market, especially towards the CE allocation bundle. In particular, the MRS estimates for the final allocations of aggregated buyers and sellers (or an average over the set of buyer and sellers, respectively) in each round are reported. Order/trade sizes and trade counts are reported to capture a sense of market participation and aggression, while also giving an idea of potential speed of convergence.

The two estimates of efficiency of interest here are allocative efficiency and distance efficiency. Allocative efficiency, here, is a general-equilibrium adaptation of the standard measure of allocative efficiency; specifically, the measure reported here is the sum of gained utilities across all traders divided by the sum of expected gain utilities (giving a reasonable analog to the relative surplus gain studied in the original definition). Distance efficiency, a new measure, makes use of the distance measure proposed in Gjerstad (2013), which is the average Euclidean distance between an allocation and the CE allocation for each trader (where the numeraire distance is normalized by the CE price). Efficiency in using this measure is defined as the difference in [distance from endowment bundle to CE bundle] and [distance from final allocation to CE], all normalized by the [distance from endowment bundle to CE bundle].[14] As the numeraire is normalized by the CE price, if a considerable amount of disequilibrium trade occurs, the distance measure, and thus efficiency measure, becomes more critical of the allocations (reducing efficiency quickly and to potentially negative values).

Finally, convergence, though already somewhat captured by the efficiency measures and MRS estimates, is examined within-round. Adjustment within-round between early prices

[14]This measure places all allocations with the same estimate on a circle around the CE bundle.

and round-end prices is captured by the statistic A^w (also borrowed from Gjerstad (2013)), which divides the sum of price deviations in the first five trades by the sum of deviations in the last five trades of a round. The speed or intensity of convergence, in the sense of reducing price deviations, is reported by examining the relationship between these summed price deviations and time-in-round.

4 Analysis

4.1 Model Performance & Investigation

		Rule Breakdowns			
Outcomes		None Enforced	Only Market	Only Behavioral	All Enforced
Prices. [CE=2.44]					
	Average Price	2.30 (1.74)	15.41 (1.81)	1.99 (0.16)	1.99 (0.20)
	Per-Unit Avg.	1.67 (0.38)	15.41 (1.81)	1.88 (0.15)	1.99 (0.20)
	$\|Price - CE\|$	1.46 (1.69)	13.06 (1.79)	0.65 (0.10)	0.77 (0.11)
	RMSE	2.00 (5.09)	15.13 (2.05)	0.77 (0.11)	0.91 (0.12)
Volume.					
	Order Size	14.96 (1.02)	1.00 (0.00)	5.76 (0.59)	1.00 (0.00)
	# Trades	18.05 (4.66)	26.03 (3.78)	30.34 (4.58)	18.85 (1.62)
	Trade Size	3.51 (0.74)	1.00 (0.00)	0.50 (0.08)	1.00 (0.00)
Efficiencies.					
	Allocative Eff.	0.65 (0.17)	0.65 (0.17)	0.83 (0.05)	0.92 (0.03)
	Distance Eff.	0.33 (0.13)	0.24 (0.09)	0.60 (0.06)	0.73 (0.06)
	Seller MRS	2.02 (0.46)	1.78 (0.24)	1.71 (0.11)	2.00 (0.12)
	Buyer MRS	3.08 (0.56)	3.65 (0.71)	3.40 (0.17)	2.99 (0.16)

Table 3: Outcome averages by treatment. Only Market is equivalent to SR:SU:OBR, while only behavioral is the same as LA:NL. Standard deviations are in ().

Table 3 reports descriptive statistics for four naturally interesting rule combinations, namely markets with no rules enforced, only market rules enforced, only behavioral rules enforced or all rules enforced (see Appendix A for an expansive table on all 32 combinations).[15] Each estimate shows the average outcome across the 250 simulations ran for each state of the model. Price estimates for the fully unconstrained markets ("None Enforced")

[15]Also see Appendix F for a visualization of the adjustments in allocative efficiency across different market and behavioral rule combinations.

are highly variable, with the average deviation being over half the magnitude of the CE price itself. Also the standard deviations on both price and price deviation are massive relative to their magnitude. Per-unit average price significantly undershoots the CE prediction, falling nearly a unit under the estimate consistently.[16] Given the disparity between average price and the per-unit average, successful orders are increasing in size at greater than a one to one rate when decreasing price. Allocative efficiency comes in far below those estimated in the orientation closest to GSS ("All Enforced"), with the average of 0.65 for "None Enforced" sitting much closer to the performance of the unconstrained ZI agents of Gode and Sunder (1993).

Introducing only market rules seems to create two hindrances to market performance. As orders (and thus trades) are restricted to be unit-quantity, price variation is determined via a uniform draw between 0 and the slope of the line from a trader's current allocation and the allocation holding either all Y in the market and one less unit of X or no Y and one more unit of X. This places much higher probability on more extreme prices compared to markets where traders have access to the full set of feasible reallocations, as is realized in the average price measures which are roughly six times the expected level. Additionally, given the restricted trade size, increased trade frequency is likely needed to accommodate the excess demand existing at the start of the market round. However, both a spread reduction rule and an order-reset rule have slowing qualities, leading to low trade volume. Given the expected trade volume in the market is roughly 20.8 units, the 26 units traded on average in "Only Market" markets seems too low (given the high price level and volatility) to reach efficient reallocations.

The "Only Behavioral" markets, however, perform markedly better. Price estimates are far less volatile, with average absolute deviation being just over half of a unit, and RMSE being a third of that reported for markets with no rules enforced (and roughly 20

[16]Price (or average price) here refers to the average price when treating each trade as equal (ignoring trade volume), while per-unit average price is the total y traded in a period divided by the total x traded in the period.

times less than those using only market rules). Both measures of efficiency are considerably higher, though average traded volume (roughly 15 units with 30 orders traded at half of a unit on average) still leaves average final allocations somewhat short of the contract curve. Enforcing all rules improves upon these results, with allocative efficiency reaching 0.92 on average, distance efficiency another 0.13 units higher than "Only Behavioral" markets, and aggregate agent MRS measures giving the tightest window around the CE price of the four combinations. Given the noticeable improvement in price, allocation and efficiency measures, the behavioral rules seem to impart traders with more intelligence than would be considered 'zero'.

By comparing 'None Enforced' to 'Only Behavioral' and 'Only Market' to 'All Enforced', adjustments from increasing trader intelligence (via individual rationality, NL, and a thinner price distribution, LA) with and without market rules enforced can be explored. Under the enforcement of no market rules, behavioral rule enforcement reduces price deviation by over half, and RMSE to nearly a third of its magnitude without. Allocative efficiency improves by 0.18 and distance efficiency nearly doubles, while trades are more frequent and both orders and trades occur at much smaller quantities. When all market rules are already enforced, efficiencies rise to an even greater extent from increasing trader intelligence through NL and LA rule; allocative efficiency increases by 0.27 and distance efficiency by 0.49. Trade counts fall, but trades are filled at prices much closer to the CE price. As such both price deviation and RMSE plummet. In either case, behavioral rule enforcement, and thus more than zero intelligence, improve market performance above what the market itself can naturally provide.

An interesting followup to examining the market performances of markets with rules clustered by type is isolating each of the rules and testing them individually. Table 4 reports estimates of the same set of outcomes for markets where only one rule is enforced. First, a no-loss constraint performs as expected in prices, with averages sticking close to the CE price, and both measures of volatility being quite low (comparable to 'Only Behavioral' and 'All Enforced' markets in Table 3). Given the large reduction in feasible trade prices in NL

Outcomes		Rule Breakdowns				
		Only NL	Only OBR	Only LA	Only SU	Only SR
Prices. [CE=2.44]						
	Average Price	2.42 (0.63)	3.76 (14.52)	1.55 (0.73)	15.25 (1.58)	2.00 (2.26)
	Per-Unit Avg.	2.38 (0.60)	1.78 (0.48)	1.11 (0.13)	15.25 (1.58)	1.63 (0.30)
	$\|Price - CE\|$	0.64 (0.37)	2.91 (14.52)	1.58 (0.70)	12.81 (1.58)	1.19 (2.23)
	RMSE	0.70 (0.39)	6.72 (62.77)	3.08 (7.78)	14.18 (1.74)	1.78 (10.29)
Volume. [CE = 5.2 p.t.]						
	Order Size	15.30 (0.88)	14.78 (1.00)	40.28 (343.31)	1.00 (0.00)	8.94 (1.42)
	# Trades	2.02 (1.17)	13.88 (3.33)	139.22 (9.47)	27.04 (4.43)	24.38 (4.83)
	Trade Size	3.49 (1.39)	3.75 (0.89)	2.25 (0.22)	1.00 (0.00)	3.26 (0.56)
Efficiencies.						
	Allocative Eff.	0.35 (0.19)	0.68 (0.16)	0.67 (0.18)	0.64 (0.17)	0.66 (0.16)
	Distance Eff.	0.26 (0.15)	0.33 (0.13)	0.15 (0.16)	0.24 (0.09)	0.36 (0.13)
	Seller MRS	1.32 (0.28)	1.97 (0.42)	2.09 (0.57)	1.77 (0.24)	2.05 (0.45)
	Buyer MRS	4.37 (0.67)	3.12 (0.54)	3.08 (0.61)	3.70 (0.69)	3.04 (0.54)

Table 4: Outcome averages by treatment for each rule in isolation. () report standard deviations. [] denotes competitive equilibrium estimates; the CE price is 2.44 and the CE trade volume is 5.2 units of X per trader (p.t.).

markets versus those not imposing NL, the average number of trades is one of the lowest among all rule-combinations (see Table A.1). As a result, efficiency levels appear to be quite low; though, at an efficiency-unit-per-trade level, the performance is much more impressive.[17] Relative to the 'None Enforced' markets, imposing only an orderbook reset rule essentially slows trade (five fewer trades with no change in trade or order size) and does not allow prices to develop long enough to improve in either volatility or price level (in fact both are worse).

Giving traders the sophistication to chose price via an angle-choice process leads to high trade counts and low average price measures. These can both be explained by the distributional implications of the process when converting to prices (see Appendix C). Single Unit markets, much like in 'Only Market' markets, drastically increase price volatility and levels. Allocative efficiency is on par with the other rules in isolation, though the distance and MRS estimates suffer due to the allocation path being much steeper than the CE path. Finally, markets imposing a spread reduction rule show little difference in performance compared to 'None Enforced' markets, with nearly identical per-unit average price, efficiency (both) and

[17]Increasing the round length to 3000 entries and 10000 entries results in 5.5 trades and 6.2 trades on average, resulting in efficiency levels of 0.83 and 0.89. At much higher entry counts, it would be expected that NL markets would reach the contract curve via the slow collapse of the lens to a point, and thus also report efficiency values very near to 1.

MRS values. As mentioned with the NL markets, it may be the case that longer rounds could tease out larger differences across rule combinations or help accommodate slower markets (justifying potentially poor performances in the standard runs); such results can be found in Appendix H.1 with estimates for 1000-entry rounds reported. Though, it should be noted, while longer runs point towards performance in the limit, these runs are still finite, with each period having finite bids and asks and thus inefficiency and final allocations off of the contract curve.[18]

Prices.

	Rule-Combo Ranks	
Outcomes	Best	Worst
Average Price	**NL** 2.42 (0.63)	*SR:SU:OBR* 15.41 (1.81)
Per-Unit Avg.	*OBR*:**NL** 2.39 (0.62)	*SR:SU:OBR* 15.41 (1.81)
$\lvert Price - CE \rvert$	*SR*:**NL** 0.46 (0.20)	*SR:SU:OBR* 13.06 (1.79)
RMSE	*SR*:**NL** 0.53 (0.23)	*SR*:**LA**:*OBR* 42.16 (314.01)

Table 5: Best and worst rule-combinations for price outcomes. **Bolded** rules are behavioral and *italicized* rules are market-oriented. Estimates and (standard deviations) reported.

Efficiencies.

	Rule-Combo Ranks	
Outcomes	Best	Worst
Allocative Eff.	*SR:SU*:**LA:NL** 0.94 (0.03)	*OBR*:**NL** 0.34 (0.18)
Distance Eff.	*SR:SU*:**LA:NL** 0.75 (0.06)	*SR*:**LA** 0.12 (0.17)
Seller MRS	*SR*:**LA** 2.10 (0.55)	*SU:OBR*:**NL** 1.22 (0.15)
Buyer MRS	*SR:SU*:**LA:NL** 2.93 (0.15)	*SU:OBR*:**NL** 4.80 (0.39)

Table 6: Best and worst rule-combinations for efficiency outcomes. **Bolded** rules are behavioral and *italicized* rules are market-oriented. Estimates and (standard deviations) reported.

In addition to investigating rules in isolation and type-clustered markets, best and worst performing rule-combinations are reported. Tables 5 and 6 identify which combinations rank first or last in price and efficiency measures. In price space, one rule has a clear advantage in approaching the CE price and maintaining low price volatility: NL. On the other hand, SR, OBR and SU are all associated (when combined) with poor price performance. As earlier analysis suggests, the single-unit rule is generating the higher prices, while SR and OBR slow

[18]This is particular to NL markets (non-NL markets need not tend to efficient allocations at the run level, but only on average), and explains why the theoretical predictions in Hurwicz et al. (1975) don't come to fruition even in the longer runs.

the market. In terms of efficiency measures, again a trend appears: the angle-choice process catalyzes reallocation more efficiently.[19] NL (when not paired with LA) is associated with low estimates, though this is again driven by low trade count. NL and LA paired together lead to the highest outcomes as NL's price performance helps adjust the direction of LA's allocation path, which due to high trade counts already approached optimal allocations on its own. As with the ZI-C in partial equilibrium, behavioral rules here (and particularly the individual rationality imposed by NL) drive improvements in price performance, as opposed to the market (rules) itself.

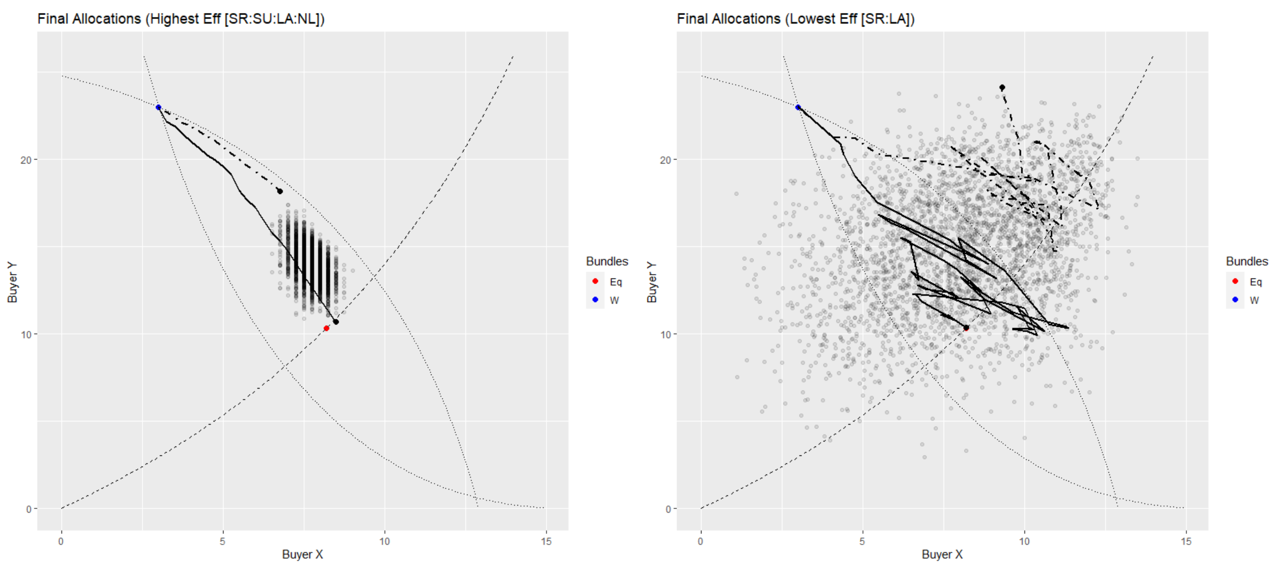

Figure 5: Final allocations for aggregated agents in SR:SU:LA:NL (left) and SR:LA (right) markets. Light grey dots denote final allocations. Dotted lines show the starting lens while the dashed line shows the set of Pareto optimal allocations (the portion within the lens represents the contract curve). The solid black line shows the best allocation path in each set of markets, while the dashed-dotted line shows the worst path.

Figure 5 gives a snapshot into convergence of the simulations in allocation-space. Aggregated agent final allocations are plotted relative to the market's starting lens and set of Pareto optimal allocations. The left panel shows such allocations for SR:SU:LA:NL (or markets with only the orderbook reset rule not enforced), while the right panel does so for SR:LA markets; these are the highest and lowest averaging market orientations for the dis-

[19]Given the analysis in Appendix C, this may is likely related to the low CE price.

tance efficiency statistic. Final allocations cluster in a small group that reaches just short of the contract curve in a relatively consistent manner. The best and worst allocation paths from the SR:SU:LA:NL sessions also show minute price adjustment across trades, and no churn (or buying and selling back and forth, which would appear as oscillation towards and away from the contract curve). Both of these suggest convergent tendencies in the allocation paths. SR:LA markets, on the other hand, are scattered throughout the Edgeworth box, often falling well outside the initial lens. Additionally, whether in the best or worst allocation path in the bunch, a significant amount of oscillation appears. Clearly, varying the rule-orientation has large consequences in both final allocations and within-round allocation adjustment. In particular, enforcing a no-loss constraint ensures (1) a final allocation within the initial lens, and (2) more direct allocation paths (as price adjustment necessarily reduces as more trades occur).

	Dependent variable:						
	Per-Unit Avg.	$\|Price - CE\|$	# Trades	Seller MRS	Buyer MRS	Alloc. Eff.	Dist. Eff.
Spread Red. (SR)	0.097***	0.060	0.505***	0.096***	−0.184***	0.057***	0.044***
	(0.021)	(0.216)	(0.110)	(0.002)	(0.004)	(0.001)	(0.001)
Single Unit (SU)	3.970***	2.135***	−8.213***	−0.068***	0.211***	0.053***	0.077***
	(0.021)	(0.216)	(0.110)	(0.002)	(0.004)	(0.001)	(0.001)
Lattice/Angle (LA)	−4.263***	−2.230***	43.468***	0.275***	−0.697***	0.255***	0.186***
	(0.021)	(0.216)	(0.110)	(0.002)	(0.004)	(0.001)	(0.001)
OB Reset (OBR)	0.074***	0.981***	−11.566***	−0.055***	0.070***	−0.005***	−0.018***
	(0.021)	(0.216)	(0.110)	(0.002)	(0.004)	(0.001)	(0.001)
No Loss (NL)	−2.308***	−4.506***	−41.880***	−0.347***	0.573***	−0.053***	0.171***
	(0.021)	(0.216)	(0.110)	(0.002)	(0.004)	(0.001)	(0.001)
Round	−0.001	−0.009	−0.001	−0.001**	0.001	−0.0001	−0.00000
	(0.003)	(0.031)	(0.016)	(0.0003)	(0.001)	(0.0001)	(0.0002)
Constant	4.996***	5.081***	43.519***	1.839***	3.499***	0.541***	0.166***
	(0.032)	(0.335)	(0.171)	(0.003)	(0.006)	(0.002)	(0.002)
Observations	95,473	95,473	96,000	95,469	95,473	96,000	96,000
Adjusted R^2	0.490	0.007	0.767	0.324	0.399	0.419	0.391
Note:							*p<0.1; **p<0.05; ***p<0.01

Table 7: Main effects or first differences, with a round time control added. The independent variables are treatment arm indicators.

While a full factorial analysis is informative of the incremental response to variation in the market's design, the main effect of relaxing or enforcing an assumption may be hard to discern.[20] Tables 7 and 8 show such impacts for a variety of variables in prices, efficiencies and measures of volatility. In first differences, allocative and distance efficiencies show unified trends, though with one divergent mechanism. Signs (and magnitude) match on efficiency estimates for all rules except No Loss, where a large positive coefficient for distance efficiency is met with a relatively small (yet significant) negative coefficient for allocative efficiency. This is reconciled by the lower trade count with allocation adjustments closer to the equilibrium path. Spread reduction and single unit assumptions show small, yet significant improvements in allocative efficiency while orderbook resetting, much like the no-loss constraints, leads to a mild reduction. Angle choice leads to a rather large improvement, likely driven by the more competitive order pricing and increased trade count.

Prices show consistent trends within rule, with an interesting across rule trend being put on display. Market-based rules (spread reduction, single unit, and orderbook reduction) show increases in price measures while behaviorally-based rules (no loss and lattice/angle) yield reductions. Distributional changes in prices vary wildly, however, between like-rule-type markets. A massive redistribution, or flattening, occurs with single-unit constrained markets, while orderbook resetting markets see a small shift right in price (as shown in Figure E.1). Markets with traders who either use an angle choice process for their orders or follow a no-loss constraint report nearly identical adjustments in average round-average price, over halving the estimates from just below 6 units of y per unit of x, to relative prices just over 0.2 units above the CE prediction. Distributional changes, however, are quite different between these two sets of markets. Figure E.1 shows the mass of the distribution funnelling quite close to the mean for angle-choice markets, however the support for the distribution remains unchanged. No-loss constrained markets also show tighter mass near the CE price. The massive reduction in the size of the support is perhaps the more remarkable result when

[20]See appendix A for such an analysis.

giving traders the intelligence to always obey their own preferences.

	Dependent variable:						
	Per-Unit Avg.	$\|Price - CE\|$	# Trades	Seller MRS	Buyer MRS	Alloc. Eff.	Distance Eff.
Spread Red. *(SR)*	0.063***	0.151	0.571***	0.123***	−0.257***	0.079***	0.058***
	(0.023)	(0.482)	(0.113)	(0.004)	(0.007)	(0.002)	(0.002)
Single Unit *(SU)*	10.601***	8.300***	−4.896***	−0.335***	0.693***	0.022***	−0.028***
	(0.023)	(0.482)	(0.113)	(0.004)	(0.007)	(0.002)	(0.002)
Lattice/Angle **(LA)**	−3.305***	−0.553	90.390***	0.016***	−0.012*	0.092***	−0.099***
	(0.023)	(0.482)	(0.113)	(0.004)	(0.007)	(0.002)	(0.002)
OB Reset *(OBR)*	0.078***	2.401***	−18.510***	−0.032***	0.008	0.038***	0.009***
	(0.023)	(0.483)	(0.113)	(0.004)	(0.007)	(0.002)	(0.002)
No Loss **(NL)**	−2.208***	−3.954***	−33.017***	−0.653***	1.099***	−0.200***	0.050***
	(0.023)	(0.485)	(0.113)	(0.004)	(0.007)	(0.002)	(0.002)
SR:*SU*	0.193***	0.268	0.133	−0.074***	0.159***	−0.052***	−0.047***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
SR:**LA**	−0.155***	−0.128	−3.811***	−0.098***	0.243***	−0.083***	−0.061***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
SR:*(OBR)*	0.036*	−0.363	0.736***	−0.026***	0.040***	−0.003**	−0.003**
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
SR:**(NL)**	0.033	0.082	2.809***	0.147***	−0.299***	0.095***	0.084***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
(SU):**(LA)**	−7.584***	−7.853***	−26.139***	0.330***	−0.885***	0.149***	0.355***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
(SU):*(OBR)*	−0.004	−1.829***	12.348***	0.074***	−0.089***	−0.017***	0.008***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
(SU):**(NL)**	−5.832***	−2.870***	7.024***	0.209***	−0.151***	−0.018***	−0.107***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
(LA):*(OBR)*	0.131***	1.187***	−17.766***	−0.029***	0.019***	−0.011***	−0.023***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
(LA):**(NL)**	5.733***	3.482***	−46.129***	0.318***	−0.751***	0.270***	0.299***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
(OBR):**(NL)**	−0.172***	−1.840***	18.571***	−0.066***	0.153***	−0.054***	−0.034***
	(0.020)	(0.432)	(0.101)	(0.004)	(0.006)	(0.002)	(0.001)
Round	−0.0004	−0.009	0.004	−0.001**	0.001	−0.0001	−0.00000
	(0.001)	(0.031)	(0.007)	(0.0003)	(0.0005)	(0.0001)	(0.0001)
Constant	3.061***	2.583***	30.463***	2.033***	3.111***	0.610***	0.283***
	(0.022)	(0.477)	(0.112)	(0.004)	(0.007)	(0.002)	(0.002)
Observations	95,473	95,473	96,000	95,469	95,473	96,000	96,000
Adjusted R^2	0.878	0.012	0.951	0.439	0.576	0.585	0.720
Note:							*p<0.1; **p<0.05; ***p<0.01

Table 8: Treatment effects including second order interactions. Treatment labels are typographed to correspond to rule orientation. Market-oriented rules are italicized (i.e., SR, SU, and OBR) and behavior-oriented rules are bolded (LA and NL).

A check of second order treatment pair impacts (without concern for higher order pairings), as shown in Table 4, reveals stark contrasts for single treatment adjustments within pairs (i.e. the interaction effects). To begin, the average price deviation within SU markets is shown to have been largely weighed down by those which also employed an angle choice process (LA=1), implying the combination of a set step size and a price choice distribution heavily skewed to favor lower prices (and thus closer to the equilibrium price) greatly abates the pattern of large price deviations seen in non-angle-choice SU markets. A similar story can be seen with buyer and seller MRS in SU markets. Pairing the distance restriction that is SU with a choice restriction on prices provides significant guidance for the ZI traders. Within SU markets, those with LA=1 entirely reverse and even improve the negative impact SU has on MRS spreads. In fact, pairing SU with either of the behaviorally-oriented rules (LA and NL) improves any measure associated with allocations.[21] The mirror to these adjustments show similar trends as well, with LA markets finding SU and NL pairings making up most of the benefit found in LA estimates from Table 3. In fact, LA market impacts on efficiency relative to non-LA markets reverse in some cases when accounting for interaction effects. Perhaps the most striking pairing is LA:NL, reporting 0.27 and nearly 0.3 unit improvements in allocative and distance efficiency respectively. The spread between aggregate agent MRS's reduces as seller (buyer) MRS rises (falls) significantly, indicating final market allocations closer to the equilibrium path and final CE bundle.

More generally, within treatment types as defined by a single rule, splitting the markets by a rule of a different type (i.e. splitting markets that are characterized by a market-oriented rule by a behaviorally-oriented rule) shows far better performance in those with both rules enforced as opposed to one. This is less true for pairings of the same type (e.g. LA:NL or SU:OBR). A finer look into the different treatments (looking at the full five factor combination) can be found in Appendix A.

[21] Pairing NL with SU, on average, proves to be a detriment to price estimates when not further parsing the markets into smaller treatment groups. Comparatively, the pairing with LA proves to be more beneficial in the allocation measures, despite both pairings yielding positive changes.

4.2 A Further Look at Prices and Efficiencies

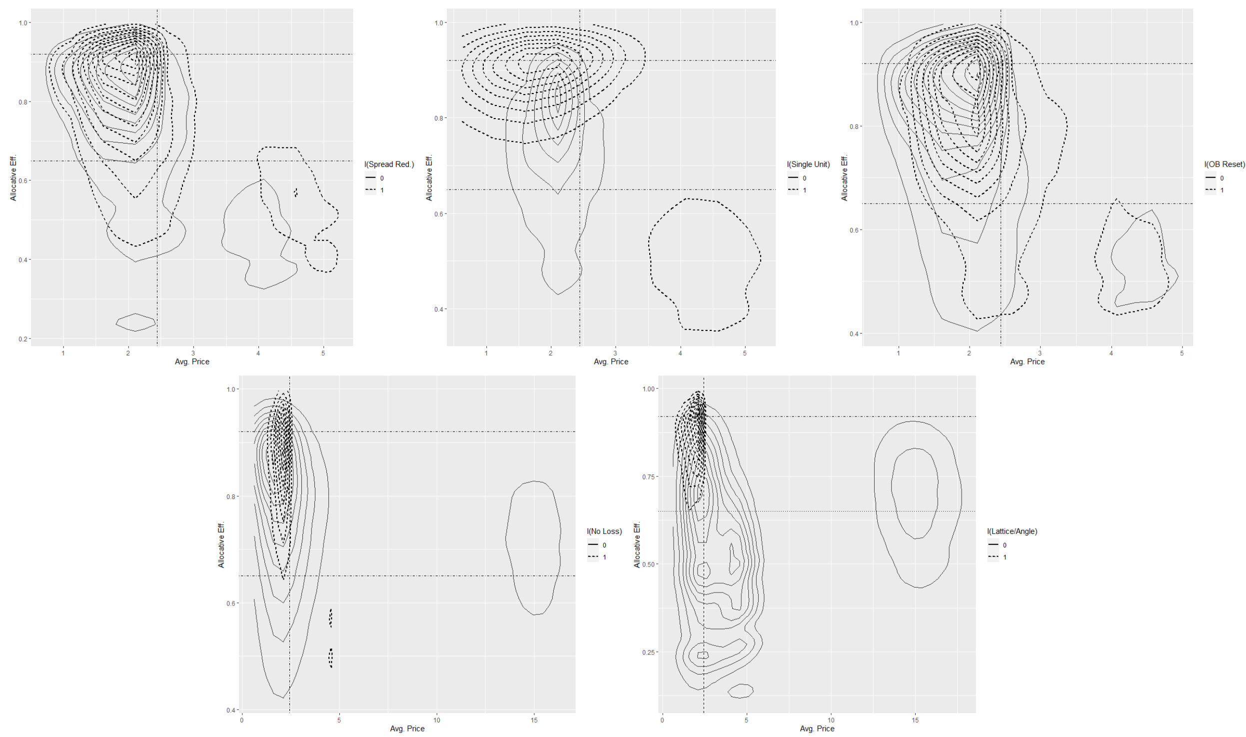

Figure 6: Bivariate densities over average price and allocative efficiency. Densities are separated based on inclusion or exclusion of each rule. The upper horizontal dotted line shows the allocative efficiency of the 1:1:1:1:1 markets; the lower horizontal dotted line shows the allocative efficiency of the 0:0:0:0:0 markets; the vertical dotted line shows the competitive equilibrium price at endowment.

Figure 6 presents bivariate densities over allocative efficiency and round-average price for each rule switch.[22] A few trends appear upon inspection. Firstly, the density shape and location match remarkably closely within rule type (with the top row showing market rules and the bottom showing behavioral rules). Secondly, within market rules, only SU reveals stark differences in distributional shape, with the allocative efficiency axis providing the majority of the transformation. The behavioral rules, however, both show markedly large reductions in distributional footprint. Both NL and LA show condensed supports on both axes, of approximately the same size. Thirdly, only NL and LA reflect the ability

[22] Appendix G reports similar distributions with per-unit averages replacing average trade price. For both Figure 6 and 7, outliers with round-average price over 50 (roughly top 0.7%) are omitted.

to fit (almost) entirely within the bounds of the two main ZI model's efficiency estimates and stay relatively close to the equilibrium price throughout the density. Clearly, on their own, enforcement of the behavioral rules proves particularly heavy-handed in the guidance of market performance.

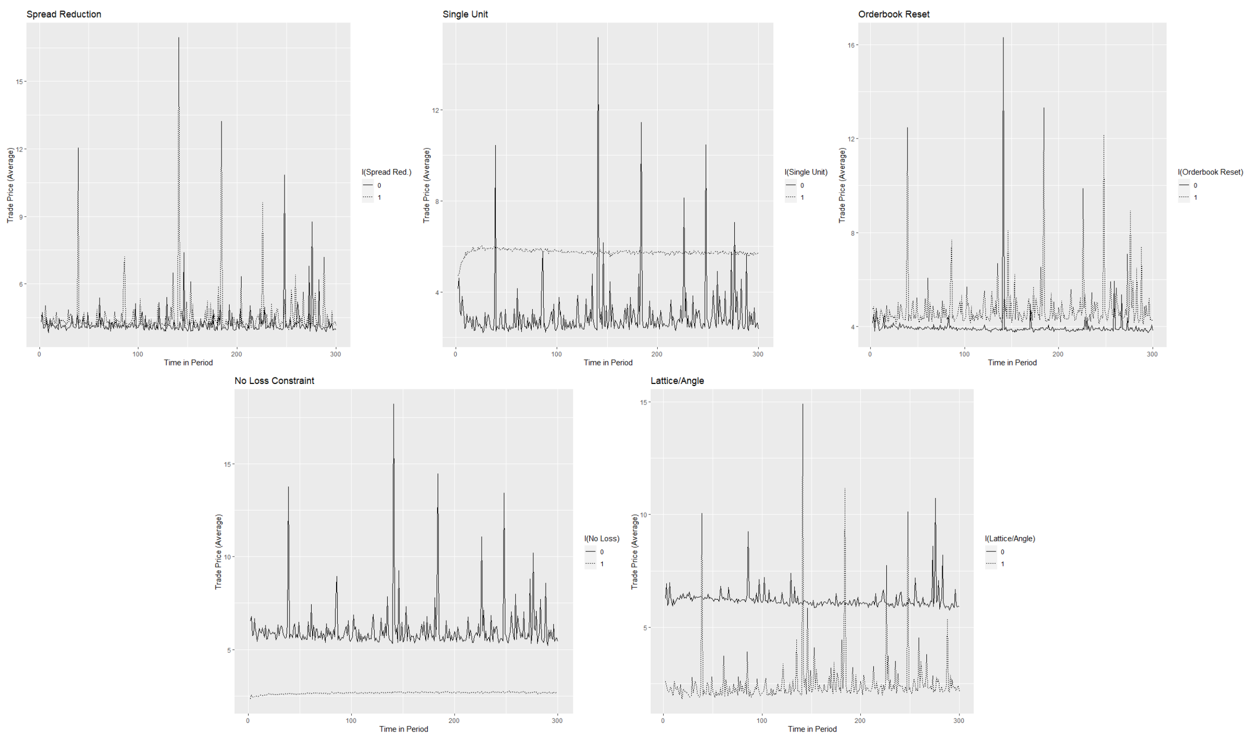

Figure 7: Time series of binned averages at the one 'second' level. Averages are taken at each second marker over the 3000 rounds in each treatment. Then, these are averaged at each second marker across within the half of the treatments that satisfy Rule=0 or Rule=1. The dotted line shows Rule=1 binned averages and the solid line shows Rule=0 binned averages.

To target prices more directly, without reference to efficiency, Figure 7 reports binned averages for trade price over time within period.[23] By definition, the floors of each of the time series plotted reveal a slope of zero, as no learning takes place in ZI markets. Volatility in trade price and the level of the trend's floor may vary however, and indeed do so in the figure. Much like Figure 6, SR and OBR report little of interest. Both show matching floors (well above CE price, but equivalent regardless of rule indicator value), though OBR markets

[23]All periods within a run where treated equally and without regard for order as no information is carried over between periods and ZI agents do not have the capacity to learn across rounds (in any treatment included in this study).

do show far more price variation than non-OBR markets. SU markets show stabler binned averages than non-SU markets, but much further from the CE price. Behaviorally-oriented rules show similar floor levels to each other, with Rule=1 markets have a floor far closer to the CE price. Where NL and LA deviate is in price variation, with NL=0 and LA=1 markets revealing higher volatility and NL=1 and LA=0 markets appearing more stable.

4.3 Convergence

In Gode and Sunder (1993) and Gode et al. (2004), market performance is largely discussed in a descriptive manner, focusing on round-end measures of allocative efficiency and average price. While these markers are important in evaluating performance in a snapshot, providing a sense of distance from theoretical predictions, some confirmation or sense of stability around these estimates should be investigated as well.

It is true that the model makes no claims about information, or behavior, transferring from round to round, so across-round measures of convergence are of no use (nor should they be as this is not the purpose of the models).[24] However, within-period convergence is certainly of interest, at least in markets where behavioral rules are enforced. Without a no-loss rule or angle-choice process, traders are essentially trading at any and all prices in $\mathbb{R}_+$ just at varying speeds, either in regard to time (SR and OBR) or in volume (SU). Given the strong distributional influences of an angle-process on price, some consistency in average price is expected, though convergence (or reduction in average deviation from said price across time) is less immediately clear.

Table 9 provides some insight into the price convergence (or lack thereof) in zero intelligence markets. With respect to prices, Table 9 examines estimates and counts of reductions in five-trade price deviations, or the rolling sum of deviations in trade price from CE price for a five trade moving window. First, A^w provides a snapshot of the start-versus-end-of-round

[24]As mentioned in Section 4.2, the lack of an upward or downward trend in binned average prices confirms this.

Within-round Price Convergence.

	None Enforced	Only Market	Only Behavioral	All Enforced
A^w	-0.09	-0.01	0.48	0.54
	(0.76)	(0.41)	(0.30)	(0.19)
Constant	1.9	4.18	1.7	1.86
	(0.53)	(0.33)	(0.43)	(0.31)
Beta	-0.04	-0.02	-0.26	-0.31
	(0.26)	(0.14)	(0.18)	(0.16)
P	0.23	0.27	0.08	0.06
	(0.29)	(0.0)	(0.19)	(0.17)
A [Min, Max]	[-9.67, 0.99]	[-3.39, 0.78]	[-2.92, 0.98]	[-0.84, 0.95]
# A>0.5	493	172	1734	1937
# Beta<0***	471	439	2026	2165
Observations	3000	3000	3000	3000

Table 9: Average estimates and counts for within-round price convergence. A^w is 1 minus the sum of price deviations from CE in the final 5 trades divided by the sum of deviations from CE in the first 5 trades of a round. Constant, Beta and P are from regressing $log(\sum_t^{t+4}(p_t - p_{CE}))$ on log(time in round). The final two rows show counts for number of rounds with $A^w > 0.5$ and number of rounds with a coefficient estimate that is statistically significantly negative. All estimates asides from the counts are means at the round level. () report standard deviations.

price deviation improvement (or lack thereof); A^w is defined as one minus the five-trade price deviation sum in the first five trades divided by the same statistic for the final five trades of a round.[25] As such, values are bounded above by 1 (assuming deviations exist in the first five trades), with 1 implying full convergence, 0 implying no improvement and negative values implying divergent price tendencies. For both 'None Enforced' and 'Only Market' rule-combination markets, A^w estimates are negative on average implying a lack of convergence. Markets with 'Only Behavioral' or 'All Enforced' rule-combinations report price deviations at the end of market rounds that are roughly half as large as those at the start of the round. The ranges for A^w shed light on how extreme some of the divergent trends can be, while markets enforcing behavioral rules (and especially all rules) show much tighter distributions of estimates that are have more mass towards the convergent values (i.e. 1). Stronger examples of convergence (such as values above 0.5) are far more common

[25]This statistic is borrowed from Gjerstad (2013).

in markets enforcing behavioral rules as well. Moving from 'None Enforced' to 'Only Behavioral' or 'Only Market' to 'All Enforce' suggests increasing trader intelligence while holding market rule enforcement constant results in an improvement in average A^w of just under 0.6 and a reduction in the lower bound of the A^w range to roughly a third of its magnitude with no behavioral rules enforced.

The other half of Table 9 reports average regression outcomes when regression the logarithm of five-trade rolling sums of price deviation from CE on the logarithm of time in market round. Convergent trends are associated with negative beta values. Average betas are more clearly negative in behavioral-rule-enforcing markets than those enforcing only market rules or no rules at all. Over two thirds of markets with either all rules enforce or only behavioral rules enforced have a negative beta significant at the 0.01 level, while less than one sixth of the markets with no rules or only market rules satisfy the same condition. As with the previous analysis, convergence metrics indicate that the behavioral rules seem to impart more than zero intelligence to the traders; or, in other words, zero intelligence again appears to not be enough.

4.4 Experimental Applications

Zero intelligence simulations have frequently provided a benchmark for laboratory market outcomes. As portrayed by the simulation exercise in this paper, however, using the ZI framework 'as-is' may not provide a reasonable comparison. Instead, this paper suggests the rules governing the simulation environment should match that of the experiment.

For example, an experiment which enforces a spread reduction rule and single unit, but allows long-lived orderbooks, should consider using SR:SU and SR:SU:NL simulations as benchmarks for an institutional lower bound and a marker for utility-improving random play, respectively. Note, however, that an angle choice rule is not recommended for benchmarking, as (1) the choice process is unnatural in terms of real-world trading, and (2) the process is highly sensitive to parametrization, especially as the equilibrium price rises away from 1.

		Gjerstad (2013)	SR:SU	SR:SU:NL	SR:NL
Allocative Efficiency.					
First Period	Mean	0.94	0.86	0.94	0.91
	S.D.	0.02	0.08	0.06	0.05
	[Min, Max]	[0.90, 0.97]	[0.63, 0.98]	[0.61, 1.00]	[0.70, 0.99]
All Periods	Mean	0.98	0.87	0.94	0.91
	S.D.	0.02	0.07	0.06	0.05
	[Min, Max]	[0.90, 1.00]	[0.55, 0.99]	[0.61, 1.00]	[0.64, 1.00]
Price & MRS.					
First Period	Average Price	93.05 (35.85)	1250.30 (190.20)	993.65 (218.25)	85.76 (23.39)
	Buyer MRS	102.59 (23.92)	411.06 (419.00)	259.92 (175.01)	68.15 (40.83)
	Seller MRS	83.04 (27.40)	191.46 (51.54)	205.00 (28.49)	158.53 (89.09)
All Periods	Average Price	94.31 (22.80)	1258.43 (188.67)	984.15 (216.84)	86.61 (24.18)
	Buyer MRS	115.37 (33.04)	405.60 (460.05)	260.61 (156.48)	72.13 (60.22)
	Seller MRS	77.13 (20.76)	195.40 (52.80)	203.82 (27.68)	158.45 (93.55)

Table 10: Gjerstad (2013) allocative efficiency outcomes compared to ZI benchmarks in the upper panel. Outcomes are at the round level, with 96 rounds across 7 sessions comprising the full data set. Each of the ZI settings were simulated 250 runs with 12 periods per run. In the lower panel, price outcomes are reported, with () denoting standard errors.

Gjerstad (2013) presents an example of an experimental market run with SR and SU rules enforced[26]; the simulated allocative efficiency benchmarks for ZI markets of the same parametrization would be 0.94 and 0.87 with and without a no-loss constraint enforced, as shown in Table 10. Regarding allocative efficiency, the human subject markets outperform both ZI benchmarks, with all 96 rounds reporting estimates above 0.87 and only three falling below 0.94.[27] Estimates for the first periods of the human sessions seem more in line with the SR:SU:NL estimates, though only two of the first rounds still fall below the ZI mean. Appendix H.3 presents an expanded example for the experiments from Gjerstad (2013), with

[26] In the experimental CDA run in Gjerstad (2013), all orders are restricted to having a quantity of 1, and a spread reduction rule is enforced in all sessions/periods (see section 3 of Gjerstad (2013), pages 468-469, to find the full CDA structure used, as well as mention of both rules above).

[27] As mentioned earlier, when run for exceptionally long run-times, NL market efficiencies will tend to 1, with a lower bound in the limit being that of the lowest efficiency allocation along the contract curve within the starting lens. Thus, the human markets finishing so high relative to moderate-length runs, and comparable to exceptionally long runs, suggests the subjects' learning process in both price and allocations outpaced random orders which follow IR.

tables matching Table H.4 for 100 simulations of with Gjerstad's parametrization, as well as with a normalized version of said parametrizations which adjusts the CE price to near 1 (see Table H.5). As can be seen in these tables as well as Table 10, prices and allocation paths can be sensitive to the experiment's parametrization. The combination of the corner endowment[28], large scaling difference between the two goods (100 times more units of Y than X) and preference parametrization of Gjerstad (2013) make for a clear example of both the sensitivity that can exist and the difference in intelligence between human subject's learning processes and zero intelligence agents. The final column depicts a setting which performs much closer to the experimental data in terms of price, namely SR:NL; by removing the single unit restriction prices are able to get closer to the CE price of 91, though at the cost of moving away from the data slightly in terms of allocative efficiency. Additionally, stark differences in the performance with and without an angle-choice process in Tables H.4 and H.5, especially in prices, portray how using such a choice process for benchmarking may be misleading.

Rule-matched simulations can also provide a more proactive benefit in the experimental design stage, as parametrizations can be tested for various desired characteristics. These could range from width or shape of size of the starting lens, different starting endowments, or even scenarios with multiple equilibria. Adjusting each of these features would change simulated behavior and hence inform researchers on trends (in terms of lower bounds) such as ease or speed of convergence, variation in anticipated allocative efficiency or buyer versus seller adjustment and surplus. Additionally, other rules that experimenters may be interested in that are not included in this paper may be easily incorporated into the simulation process; for example, imposing spread reduction at the trader level[29] or using alternative order-time orderings (e.g. frequent batching). In any application of this model in this manner, however,

[28]With the preferences used in Gjerstad (2013), this endowment means essentially the entire contract curve is within the starting lens, implying that even in NL settings, early trade can be highly volatile.

[29]This would mean a trader only replaces their existing order if their new order is at a better price, but at a price that may not be the best overall of that order type.

the simulations should be taken as a lower bound, as certain outcomes can be sensitive to the potential design choices discussed above. If the experimenter's intent is to instead predict laboratory behavior, a minimal-intelligence model (as opposed to zero-intelligence) or a version of this model with even more market and/or behavioral rules may be more appropriate.

5 Conclusion

Understanding both the implications of the rules implied by a market institution and the underlying behavior defining trader actions in said market are crucial tasks in economic research. The main issue plaguing such an endeavor is the entanglement between the two. One way to isolate the first study is to place traders with no strategic behavior in the market, so as to let market outcomes be only guided by the rules of the institution. Gode and Sunder (1993) proposed such a model and test in a partial equilibrium setting, and then brought the model to a general equilibrium setting via the Edgeworth box (Gode et al., 2004). The proposed zero intelligence traders, however, either abode by potentially influential behavioral assumptions (such as a no-loss constraint, or an order choice process giving more weight to less aggressive prices), or participated in markets with rules that may guide the allocation path. This paper (1) summarizes and provides adjustments to the key assumptions of ZI-GE, and (2) conducts a test of the assumptions made in this model and those aforementioned.

I test the major assumptions made in ZI models, as well as those made in the models' respective CDA markets, via a novel, expansive simulation procedure. All combinations of the five rule variants either relaxed or enforced are simulated with traders from this paper's model. Each variation was simulated 250 times with each run containing 3600 entries, yielding a data set of 28.8 million market entries and order placements across 96,000 trading periods in 8000 simulated markets. First differences show an improvement across the board when imposing one of the five rules, as well as a reorganization of gains from trade resulting in systematic improvements in distance efficiency. An interactions model reports the incremental impact of these assumptions in the full factorial design.

Estimates for key price and allocation outcomes suggest that 'zero intelligence' is indeed not enough, with behavioral assumptions providing considerable guidance to market convergence and performance. Allocative and distance efficiencies are 0.26 and 0.40 units lower than those found in ZI-GE, while average price is slightly (though insignificantly) closer to CE and price volatility is twice as large in the least restricted version of the model. When paired with another assumption prone to price funneling, both spread reduction and no-loss rules provide large improvements in market performance. Interestingly, markets with either {pairs of enforced rules} or {at most one relaxed assumption} exhibit the most equilibrating tendencies. Markets in which traders' order choice behavior is dictated by an angle-choice process or a no-loss tendency benefit immensely in market performance from the added intelligence.

In addition, this paper offers a customizable testbed for establishing lower-bound benchmarks for markets in the laboratory. Matching rules in the simulations and experiments is advised when used for the purposes of benchmarking elicited data or testing various parametrizations. In either regard, using the lattice choice rule is advised as reallocation, and thus price choice, is less intelligent and perhaps more intuitive.

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Appendix A Descriptive Continued

					Outcome:										
SR	SU	LA	OBR	NL	Price	Per-Unit Avg.	$\lvert Price - CE \rvert$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.	Dist. Eff.
0	0	0	0	0	2.3 (1.74)	1.67 (0.38)	1.46 (1.69)	2.00 (5.09)	14.96 (1.02)	18.05 (4.66)	3.51 (0.74)	2.02 (0.46)	3.08 (0.56)	0.65 (0.17)	0.33 (0.13)
0	0	0	0	1	**{2.42} (0.63)**	2.38 (0.60)	0.64 (0.37)	0.70 (0.39)	15.30 (0.88)	2.02 (1.17)	3.49 (1.39)	1.32 (0.28)	4.37 (0.67)	0.35 (0.19)	0.26 (0.15)
0	0	0	1	0	3.76 (14.52)	1.78 (0.48)	2.91 (14.52)	6.72 (62.77)	14.78 (1.00)	13.88 (3.33)	3.75 (0.89)	1.97 (0.42)	3.12 (0.54)	0.68 (0.16)	0.33 (0.13)
0	0	0	1	1	**{2.42} (0.65)**	**{2.39} (0.62)**	0.64 (0.38)	0.69 (0.40)	15.34 (0.89)	1.77 (0.95)	3.65 (1.48)	1.28 (0.26)	4.45 (0.65)	**[0.34] (0.18)**	0.25 (0.14)
0	0	1	0	0	1.55 (0.73)	1.11 (0.13)	1.58 (0.70)	3.08 (7.78)	40.28 (343.31)	139.22 (9.47)	2.25 (0.22)	2.09 (0.57)	3.08 (0.61)	0.67 (0.18)	0.15 (0.16)
0	0	1	0	1	1.99 (0.16)	1.88 (0.15)	0.65 (0.10)	0.77 (0.11)	5.76 (0.59)	30.34 (4.58)	0.50 (0.08)	1.71 (0.11)	3.40 (0.17)	0.83 (0.05)	0.60 (0.06)
0	0	1	1	0	9.04 (179.29)	1.3 (0.21)	9.06 (179.29)	60.04 (1563.85)	29.66 (95.99)	78.71 (5.93)	2.13 (0.27)	2.05 (0.47)	3.03 (0.51)	0.74 (0.14)	0.15 (0.15)
0	0	1	1	1	1.96 (0.18)	1.84 (0.17)	0.80 (0.11)	0.93 (0.12)	5.79 (0.59)	24.70 (3.48)	0.53 (0.10)	1.57 (0.11)	3.64 (0.19)	0.76 (0.06)	0.52 (0.06)
0	1	0	0	0	15.25 (1.58)	15.25 (1.58)	12.81 (1.58)	14.18 (1.74)	1.00 (0.00)	27.04 (4.43)	1.00 (0.00)	1.77 (0.24)	3.70 (0.69)	0.64 (0.17)	0.24 (0.09)
0	1	0	0	1	4.06 (0.47)	4.06 (0.47)	1.66 (0.43)	1.77 (0.39)	1.00 (0.00)	3.90 (1.56)	1.00 (0.00)	1.23 (0.15)	4.77 (0.41)	0.42 (0.14)	0.24 (0.09)
0	1	0	1	0	14.99 (1.80)	14.99 (1.80)	12.62 (1.78)	14.54 (2.05)	1.00 (0.00)	23.61 (3.91)	1.00 (0.00)	1.78 (0.24)	3.67 (0.69)	0.65 (0.17)	0.24 (0.09)
0	1	0	1	1	4.04 (0.49)	4.04 (0.49)	1.66 (0.43)	1.78 (0.39)	1.00 (0.00)	3.81 (1.51)	1.00 (0.00)	**[1.22] (0.15)**	**[4.80] (0.39)**	0.42 (0.14)	0.23 (0.09)
0	1	1	0	0	1.27 (0.23)	1.27 (0.23)	1.54 (0.12)	1.77 (0.26)	1.00 (0.00)	84.3 (5.39)	1.00 (0.00)	1.96 (0.22)	3.04 (0.22)	0.87 (0.06)	0.53 (0.08)
0	1	1	0	1	1.93 (0.20)	1.93 (0.20)	0.72 (0.11)	0.84 (0.12)	1.00 (0.00)	18.43 (1.84)	1.00 (0.00)	1.96 (0.13)	3.05 (0.17)	0.91 (0.04)	0.71 (0.06)
0	1	1	1	0	1.68 (0.29)	1.68 (0.29)	1.85 (0.21)	2.44 (0.53)	1.00 (0.00)	63.91 (4.74)	1.00 (0.00)	1.95 (0.23)	3.05 (0.29)	0.84 (0.08)	0.50 (0.09)
0	1	1	1	1	2.00 (0.20)	2.00 (0.20)	0.77 (0.11)	0.90 (0.13)	1.00 (0.00)	17.46 (1.71)	1.00 (0.00)	1.91 (0.13)	3.11 (0.18)	0.90 (0.04)	0.69 (0.06)
1	0	0	0	0	2.00 (2.26)	1.63 (0.30)	1.19 (2.23)	1.78 (10.29)	8.94 (1.42)	24.38 (4.83)	3.26 (0.56)	2.05 (0.45)	3.04 (0.54)	0.66 (0.16)	0.36 (0.13)
1	0	0	0	1	2.29 (0.31)	2.26 (0.30)	**{0.46} (0.20)**	**{0.53} (0.23)**	9.66 (2.36)	4.72 (1.52)	3.18 (0.83)	1.80 (0.35)	3.37 (0.60)	0.63 (0.15)	0.48 (0.13)
1	0	0	1	0	3.78 (10.43)	1.85 (0.43)	2.84 (10.40)	7.11 (45.10)	11.79 (1.23)	16.17 (3.16)	3.73 (0.80)	2.03 (0.40)	3.04 (0.49)	0.71 (0.15)	0.36 (0.13)
1	0	0	1	1	2.33 (0.36)	2.28 (0.34)	0.52 (0.22)	0.60 (0.25)	11.29 (1.87)	3.31 (0.95)	3.77 (1.09)	1.63 (0.31)	3.66 (0.58)	0.61 (0.16)	0.46 (0.14)
1	0	1	0	0	3.17 (38.28)	1.15 (0.15)	3.24 (38.28)	18.95 (425.89)	10.04 (49.81)	127.29 (8.29)	2.19 (0.22)	**{2.10} (0.55)**	3.07 (0.72)	0.67 (0.18)	**[0.12] (0.17)**
1	0	1	0	1	2.05 (0.15)	1.93 (0.15)	0.61 (0.11)	0.74 (0.12)	1.81 (0.58)	35.38 (4.51)	0.46 (0.07)	1.82 (0.10	3.23 (0.14)	0.87 (0.04)	0.65 (0.05)
1	0	1	1	0	7.33 (36.23)	1.34 (0.23)	7.33 (36.22)	**[42.16] (314.01)**	21.27 (53.84)	74.31 (5.23)	2.1 (0.28)	2.06 (0.45)	3.02 (0.49)	0.75 (0.13)	0.16 (0.15)
1	0	1	1	1	1.94 (0.17)	1.83 (0.17)	0.83 (0.11)	0.96 (0.12)	4.01 (0.71)	26.64 (3.27)	0.50 (0.09)	1.60 (0.11)	3.59 (0.18)	0.78 (0.06)	0.53 (0.06)
1	1	0	0	0	15.31 (1.52)	15.31 (1.52)	12.89 (1.52)	14.19 (1.58)	1.00 (0.00)	29.83 (4.39)	1.00 (0.00)	1.80 (0.23)	3.62 (0.69)	0.66 (0.17)	0.24 (0.08)
1	1	0	0	1	4.63 (0.37)	4.63 (0.37)	2.21 (0.34)	2.27 (0.29)	1.00 (0.00)	4.51 (1.53)	1.00 (0.00)	1.34 (0.15)	4.54 (0.39)	0.52 (0.14)	0.30 (0.09)
1	1	0	1	0	**[15.41] (1.81)**	**[15.41] (1.81)**	**[13.06] (1.79)**	15.13 (2.05)	1.00 (0.00)	26.03 (3.78)	1.00 (0.00)	1.78 (0.24)	3.65 (0.71)	0.65 (0.17)	0.24 (0.09)
1	1	0	1	1	4.52 (0.41)	4.52 (0.41)	2.11 (0.36)	2.21 (0.30)	1.00 (0.00)	4.41 (1.55)	1.00 (0.00)	1.32 (0.15)	4.58 (0.40)	0.50 (0.14)	0.29 (0.09)
1	1	1	0	0	1.43 (0.26)	1.43 (0.26)	1.67 (0.16)	2.06 (0.40)	1.00 (0.00)	78.4 (5.00)	1.00 (0.00)	1.97 (0.21)	3.02 (0.24)	0.87 (0.06)	0.52 (0.09)
1	1	1	0	1	1.91 (0.20)	1.91 (0.20)	0.73 (0.12)	0.85 (0.13)	1.00 (0.00)	19.89 (1.75)	1.00 (0.00)	2.05 (0.13)	**{2.93} (0.15)**	**{0.94} (0.03)**	**{0.75} (0.06)**
1	1	1	1	0	1.74 (0.30)	1.74 (0.30)	1.91 (0.22)	2.55 (0.56)	1.00 (0.00)	65.09 (4.62)	1.00 (0.00)	1.97 (0.22)	3.02 (0.29)	0.85 (0.07)	0.49 (0.10)
1	1	1	1	1	1.99 (0.20)	1.99 (0.20)	0.77 (0.11)	0.91 (0.12)	1.00 (0.00)	18.85 (1.62)	1.00 (0.00)	2.00 (0.12)	2.99 (0.16)	0.92 (0.03)	0.73 (0.06)

Table A.1: Outcome averages by treatment. Observations are at the round-average or round-end level. The left panel shows the assumptions enforced. **{Bolded}** estimates are the 'best' in the column, while **[bolded]** are the 'worst'.

Appendix B Step-size Variation

	Only step-size				All Constraints			
	r=0.1	r=0.75	r=1	r=1.5	r=0.1	r=0.75	r=1	r=1.5
Per-Unit Avg. Price	0.46	0.45	0.45	0.47	1.6	1.73	1.76	1.81
	(0.05)	(0.06)	(0.06)	(0.06)	(0.13)	(0.14)	(0.14)	(0.15)
# Trades	147.67	146.55	147.25	148.71	41.09	31.4	28.83	24.9
	(10.11)	(10.33)	(10.42)	(10.14)	(4.16)	(3.03)	(2.75)	(2.3)
Seller MRS	0.88	1.48	1.64	1.84	0.87	1.33	1.46	1.67
	(0.01)	(0.12)	(0.16)	(0.25)	(0.01)	(0.06)	(0.07)	(0.09)
Buyer MRS	5.08	3.39	3.22	3.09	5.55	4.11	3.84	3.46
	(0.08)	(0.14)	(0.14)	(0.16)	(0.04)	(0.13)	(0.14)	(0.15)
Allocative Eff.	0.23	0.78	0.83	0.86	0.14	0.62	0.71	0.82
	(0.02)	(0.05)	(0.05)	(0.05)	(0.01)	(0.05)	(0.05)	(0.04)
Distance Eff.	0.12	0.39	0.38	0.35	0.07	0.38	0.46	0.58
	(0.01)	(0.03)	(0.04)	(0.05)	(0.01)	(0.04)	(0.04)	(0.05)

Table B.1: Round-average estimates for markets with different step-sizes (r). Left panel shows estimates where only enforced constraint is step-size, while right panel enforces all five constraints.

Appendix C Lattice draw vs. Radian draw

This appendix is meant to clarify differences in the reallocation likelihoods and price likelihoods associated with a uniform draw over feasible reallocations (i.e. new (x, y) bundles) and an angle-based price-choice process. In both cases, no restriction over quantities are considered to allow a visualization and assessment over the entire feasible reallocation space. I show simulated draws over ask orders, a similar exercise over bids would show comparable results.

Figure C.1 compares simulated draws using both processes when no no-loss constraint is imposed. Each plot shows 10,000 order draws, plotting each (potential) reallocation point, a binned joint density map and marginal densities over each good. Clearly, the left plot shows uniformity across the full space while the right plot shows peaked y draws near the current allocation's holdings of y. The combination of a radian draw, with a uniform distribution along vectors moving outwards from the current allocation yields a sample closer to the allocation and lower in price.

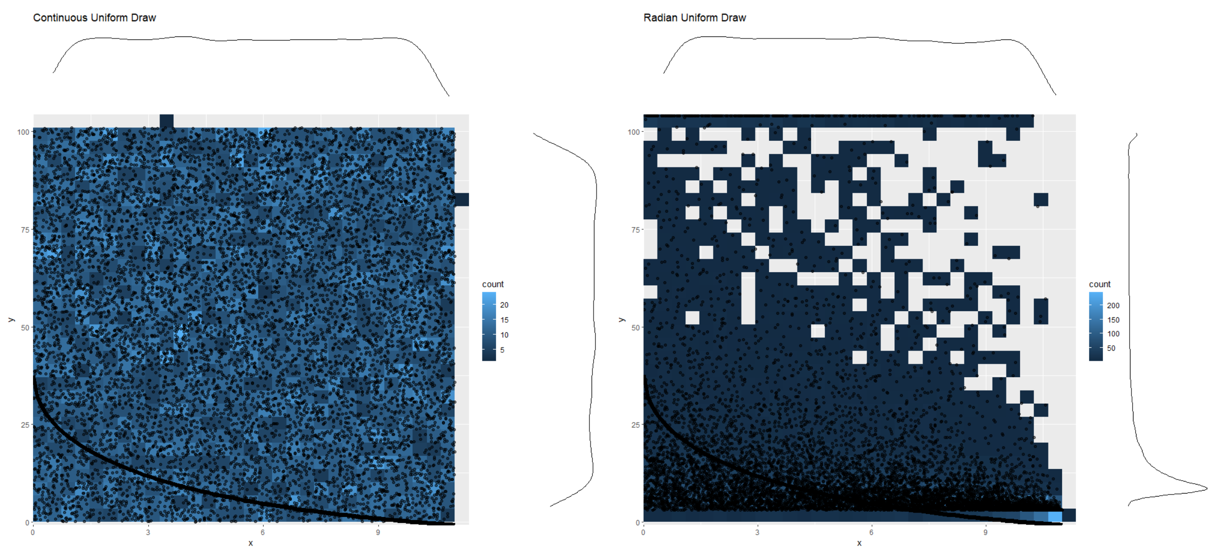

Figure C.1: **Left:** Uniform random draws of new (x, y) bundles. **Right:** Uniform draws over radians from $[0, \pi/2]$ paired with random quantity draws along the drawn price vector. In both plots, 10000 simulated order reallocations are show. Each dot represents an order, the map depicts the joint density while the curves on the borders are marginal densities.

Figure C.2 recreates this simulation exercise with a no-loss constraint enforced. Again the left plot shows a a uniform distribution over the feasible reallocations. The right plot now heavily favors reallocations closer to the trader's indifference curve. While this may seem harmful to the trader's strategy, it is actually beneficial for crossing orders. Given the set of prices that traders may trade at under a no-loss constraint is bounded by the set of traders' marginal rates of substitution, maintaining an order strategy that keeps prices close to a trader's marginal rate of substitution gives them a better shot at both crossing existing orders and placing orders with a prices other traders may like. As such, the uniform draw over reallocations can be though of as simpler, or of lower intelligence.

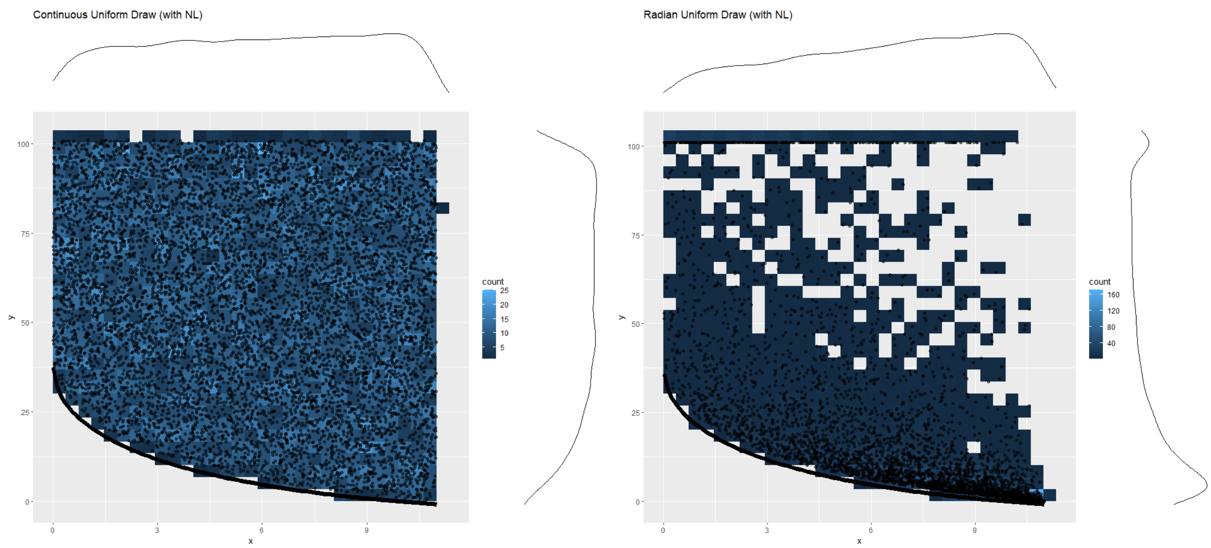

Figure C.2: **Left:** Uniform random draws of new (x, y) bundles that are utility-improving. **Right:** Uniform draws over radians from $[0, \pi/2]$ paired with random quantity draws along the drawn price vector that are utility-improving. In both plots, 10000 simulated order reallocations are show. Each dot represents an order, the map depicts the joint density while the curves on the borders are marginal densities.

Appendix D Regression Table Continued

	Dependent variable:									
	Price	Per-Unit Avg.	$\lvert Price - CE \rvert$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Spread Red (SR)	−0.307	−0.048***	−0.266	−0.215	−6.021***	6.332***	−0.245***	0.030***	−0.041***	0.011***
	(0.859)	(0.017)	(0.859)	(7.565)	(1.661)	(0.107)	(0.013)	(0.008)	(0.012)	(0.003)
Single Unit (SU)	12.946***	13.574***	11.356***	12.182	−13.964***	8.986***	−2.508***	−0.248***	0.621***	−0.005
	(0.859)	(0.017)	(0.859)	(7.565)	(1.661)	(0.107)	(0.013)	(0.008)	(0.012)	(0.003)
Lattice/Angle (LA)	−0.753	−0.562***	0.124	1.088	25.318***	121.174***	−1.259***	0.065***	−0.001	0.020***
	(0.859)	(0.017)	(0.859)	(7.565)	(1.661)	(0.107)	(0.013)	(0.008)	(0.012)	(0.003)
OB Reset (OBR)	1.453*	0.107***	1.449*	4.723	−0.180	−4.175***	0.243***	−0.051***	0.042***	0.031***
	(0.859)	(0.017)	(0.859)	(7.565)	(1.661)	(0.107)	(0.013)	(0.008)	(0.012)	(0.003)
No Loss (NL)	0.120	0.709***	−0.816	−1.300	0.333	−16.027***	−0.017	−0.705***	1.284***	−0.297***
	(0.878)	(0.018)	(0.878)	(7.728)	(1.661)	(0.107)	(0.013)	(0.008)	(0.012)	(0.003)
⋮										
Constant	2.302***	1.674***	1.457**	1.996	14.964***	18.050***	3.508***	2.020***	3.081***	0.648***
	(0.608)	(0.012)	(0.608)	(5.349)	(1.175)	(0.075)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,473	95,473	95,473	95,473	96,000	96,000	95,473	95,469	95,473	96,000
Adjusted R^2	0.017	0.978	0.014	0.002	0.020	0.986	0.834	0.467	0.601	0.608
Note:										*p<0.1; **p<0.05; ***p<0.01

Table D.1: Interaction regression results. First order effects are reported here, the rest are in tables to follow.

Tables D.1-D.3 present the treatment analysis of the full factorial design. Each of the five assumptions/rules being tested is given an indicator, $I(rule)$, with a value of 1 representing the presence of the constraint in the simulations. The estimation process is represented the following interaction design:

$$Y_i = \alpha + \sum_{i \in Rules} \beta_i I(i=1) + \sum_{i \in Rules} \sum_{j \in Rules_{/\{i\}}} \beta_{ij} I(i=1) I(j=1) + \cdots + \beta_{ijklm} \prod_{i \in Rules} I(i=1) \quad (6)$$

The main effects of the model, i.e. first summation from (6), is provided in Table D.1. Relative to Table 3, price estimates flip in sign with magnitudes falling for behavioral rules and increasing for market rules. MRS and allocative efficiency estimates are small in size with the exception of SU-only markets in MRS estimates and NL-only markets in both.

Moving to Table D.2, allocation adjustments seem to be the main beneficiary of imposing a second assumption. All significant estimates for seller MRS being positive, paired with all-but-one significant estimates being negative implies convergence in allocation space. A few act as recoveries, with the damage of orderbook resetting, the no-loss constraint and single unit orders being reclaimed by inclusions of a second constraint. Angle choice and spread reduction restrictions are especially effective in progressing NL markets to a more successful final allocation. Similarly, the vast majority of interactions reflect in an increase in efficiency. Larger improvements are reflective of reversals for NL markets mostly, while smaller improvements are most often continuations of efficiency gain in SR and LA markets.

Table D.3, which reports the quinary interaction, provides an interesting connection to the literature. As the GSS model (with single unit instead of single step orders/trades) enforces all five assumptions, flipping the signs in Table D.3 allows the coefficients to represent the average impact of relaxing a single assumption in the model. A slight tightening (∼0.17 reduction) of the MRS spread represents a small positive effect of assumption relaxation, while reductions in per-unit average price and trade count present potentially negative impacts. As Table 3 will show however, individual comparisons reveal relaxing the angle choice

	Dependent variable:									
	Price	Per-Unit Avg.	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
⋮										
SR:SU	0.365	0.107***	0.346	0.222	6.021**	−3.540***	0.245***	−0.003	−0.041**	0.006
	(1.215)	(0.024)	(1.215)	(10.698)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
SR:LA	1.923	0.081***	1.929	16.084	−24.220***	−18.269***	0.183***	−0.015	0.035**	−0.009*
	(1.215)	(0.024)	(1.215)	(10.698)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
SR:OBR	0.331	0.119***	0.204	0.608	3.031	−4.038***	0.228***	0.027**	−0.044**	0.019***
	(1.215)	(0.024)	(1.215)	(10.698)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
SR:NL	0.171	−0.078***	0.082	0.047	0.381	−3.640***	−0.068***	0.452***	−0.950***	0.271***
	(1.229)	(0.025)	(1.229)	(10.815)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
SU:LA	−13.226***	−13.417***	−11.400***	−13.495	−25.318***	−63.908***	1.259***	0.119***	−0.665***	0.204***
	(1.215)	(0.024)	(1.215)	(10.698)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
SU:OBR	−1.714	−0.368***	−1.642	−4.361	0.180	0.747***	−0.243***	0.056***	−0.078***	−0.028***
	(1.215)	(0.024)	(1.215)	(10.698)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
SU:NL	−11.307***	−11.896***	−10.342***	−11.105	−0.333	−7.105***	0.017	0.162***	−0.212***	0.079***
	(1.230)	(0.025)	(1.230)	(10.823)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
LA:OBR	6.041***	0.080***	6.025***	52.232***	−10.445***	−56.339***	−0.365***	0.011	−0.092***	0.039***
	(1.215)	(0.024)	(1.215)	(10.698)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
LA:NL	0.318	0.060**	−0.110	−1.013	−34.859***	−92.859***	−1.735***	0.334***	−0.969***	0.461***
	(1.229)	(0.025)	(1.229)	(10.814)	(2.349)	(0.151)	(0.019)	(0.011)	(0.017)	(0.005)
OBR:NL	−1.456	−0.103***	−1.450	−4.729	0.224	3.924***	−0.086***	0.013	0.047***	−0.044***
	(1.241)	(0.025)	(1.241)	(10.925)	(2.349)	(0.151)	(0.019)	(0.011)	(0.018)	(0.005)
⋮										
Constant	2.302***	1.674***	1.457**	1.996	14.964***	18.050***	3.508***	2.020***	3.081***	0.648***
	(0.608)	(0.012)	(0.608)	(5.349)	(1.175)	(0.075)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,473	95,473	95,473	95,473	96,000	96,000	95,473	95,469	95,473	96,000
Adjusted R^2	0.017	0.978	0.014	0.002	0.020	0.986	0.834	0.467	0.601	0.608
Note:										*p<0.1; **p<0.05; ***p<0.01

Table D.2: Interaction regression results for second order interactions. This is a continuation of the regression estimates in Table 2.

	Dependent variable:									
	Price	Per-Unit Avg.	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
⋮										
SR:SU:LA:OBR:NL	−2.984	0.522***	−3.079	−33.512	18.244***	6.007***	0.227***	−0.094***	0.168***	−0.003
	(3.448)	(0.069)	(3.448)	(30.349)	(6.645)	(0.426)	(0.053)	(0.031)	(0.049)	(0.014)
Constant	2.302***	1.674***	1.457**	1.996	14.964***	18.050***	3.508***	2.020***	3.081***	0.648***
	(0.608)	(0.012)	(0.608)	(5.349)	(1.175)	(0.075)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,473	95,473	95,473	95,473	96,000	96,000	95,473	95,469	95,473	96,000
Adjusted R^2	0.017	0.978	0.014	0.002	0.020	0.986	0.834	0.467	0.601	0.608
Note:										*p<0.1; **p<0.05; ***p<0.01

Table D.3: Interaction regression results for fifth order interaction. This is a continuation of the regression estimates in Table 2.

provides most of this variation.

Tertiary interactions (Table D.4) show mostly decays in market success (e.g. price and price deviation, both MRS measures, and allocative efficiency). Most, if not all, of the improvements seen in Table D.2 are reversed when adding a third assumption to the market (assuming the remaining two assumptions are relaxed). As most of the measures have one or two seemingly negatively-associated assumptions, and each assumption is enforced in six of the ten tertiary interactions, a systematic mild decay is not overly surprising. Quaternary interactions are reported in Table D.4.

	Dependent variable:									
	Price	Per-Unit Avg.	$\|Price - CE\|$	RMSE	Order Size	# Trades	Trade Size	Seller MRS	Buyer MRS	Alloc. Eff.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
⋮										
SR:SU:LA	−1.825	0.017	−1.878	−15.801	24.220***	9.580***	−0.183***	0.006	0.029	−0.009
	(1.719)	(0.035)	(1.719)	(15.130)	(3.323)	(0.213)	(0.026)	(0.015)	(0.024)	(0.007)
SR:SU:OBR	0.031	0.243***	0.152	−0.028	−3.031	3.670***	−0.228***	−0.047***	0.106***	−0.036***
	(1.719)	(0.035)	(1.719)	(15.130)	(3.323)	(0.213)	(0.026)	(0.015)	(0.024)	(0.007)
SR:SU:NL	0.337	0.586***	0.389	0.440	−0.381	1.456***	0.068***	−0.365***	0.794***	−0.195***
	(1.729)	(0.035)	(1.729)	(15.221)	(3.323)	(0.213)	(0.026)	(0.015)	(0.025)	(0.007)
SR:LA:OBR	−3.659**	−0.107***	−3.592**	−34.354**	18.825***	11.577***	−0.196***	−0.031**	0.039	−0.008
	(1.719)	(0.035)	(1.719)	(15.130)	(3.323)	(0.213)	(0.026)	(0.015)	(0.024)	(0.007)
SR:LA:NL	−1.730	0.093***	−1.790	−15.951	25.913***	20.619***	0.092***	−0.360***	0.795***	−0.232***
	(1.728)	(0.035)	(1.728)	(15.212)	(3.323)	(0.213)	(0.026)	(0.015)	(0.025)	(0.007)
SR:OBR:NL	−0.287	−0.099***	−0.139	−0.527	−1.445	2.879***	0.207***	−0.154***	0.237***	−0.030***
	(1.737)	(0.035)	(1.737)	(15.292)	(3.323)	(0.213)	(0.027)	(0.015)	(0.025)	(0.007)
SU:LA:OBR	−5.373***	0.588***	−5.518***	−51.930***	10.445***	39.372***	0.365***	−0.023	0.143***	−0.066***
	(1.719)	(0.035)	(1.719)	(15.130)	(3.323)	(0.213)	(0.026)	(0.015)	(0.024)	(0.007)
SU:LA:NL	11.532***	11.790***	10.452***	12.487	34.859***	50.120***	1.735***	0.209***	−0.092***	−0.199***
	(1.729)	(0.035)	(1.729)	(15.218)	(3.323)	(0.213)	(0.026)	(0.015)	(0.025)	(0.007)
SU:OBR:NL	1.699	0.346***	1.647	4.376	−0.224	−0.586***	0.086***	−0.030*	0.017	0.032***
	(1.739)	(0.035)	(1.739)	(15.302)	(3.323)	(0.213)	(0.027)	(0.015)	(0.025)	(0.007)
LA:OBR:NL	−6.067***	−0.126***	−5.876***	−52.067***	10.434***	50.950***	0.241***	−0.117***	0.247***	−0.094***
	(1.737)	(0.035)	(1.737)	(15.291)	(3.323)	(0.213)	(0.027)	(0.015)	(0.025)	(0.007)
SR:SU:LA:OBR	3.202	−0.350***	3.164	33.599	−18.825***	−4.125***	0.196***	0.055**	−0.110***	0.030***
	(2.431)	(0.049)	(2.431)	(21.397)	(4.699)	(0.301)	(0.037)	(0.022)	(0.034)	(0.010)
SR:SU:LA:NL	1.046	−0.776***	1.198	15.187	−25.913***	−11.079***	−0.092**	0.349***	−0.741***	0.182***
	(2.438)	(0.049)	(2.438)	(21.461)	(4.699)	(0.301)	(0.037)	(0.022)	(0.035)	(0.010)
SR:SU:OBR:NL	−0.165	−0.353***	−0.312	−0.124	1.445	−2.519***	−0.207***	0.164***	−0.282***	0.037***
	(2.445)	(0.049)	(2.445)	(21.523)	(4.699)	(0.301)	(0.037)	(0.022)	(0.035)	(0.010)
SR:LA:OBR:NL	3.540	0.034	3.601	34.337	−18.244***	−13.518***	−0.227***	0.075***	−0.116***	−0.007
	(2.444)	(0.049)	(2.444)	(21.512)	(4.699)	(0.301)	(0.037)	(0.022)	(0.035)	(0.010)
SU:LA:OBR:NL	5.487**	−0.455***	5.409**	51.815**	−10.434**	−34.867***	−0.241***	0.097***	−0.265***	0.117***
	(2.445)	(0.049)	(2.445)	(21.519)	(4.699)	(0.301)	(0.037)	(0.022)	(0.035)	(0.010)
⋮										
Constant	2.302***	1.674***	1.457**	1.996	14.964***	18.050***	3.508***	2.020***	3.081***	0.648***
	(0.608)	(0.012)	(0.608)	(5.349)	(1.175)	(0.075)	(0.009)	(0.005)	(0.009)	(0.002)
Observations	95,473	95,473	95,473	95,473	96,000	96,000	95,473	95,469	95,473	96,000
Adjusted R^2	0.017	0.978	0.014	0.002	0.020	0.986	0.834	0.467	0.601	0.608

Note: *p<0.1; **p<0.05; ***p<0.01

Table D.4: Interaction regression results for third order and fourth interactions. This is a continuation of the regression estimates in Table 2.

Appendix E Round-Average Price Densities

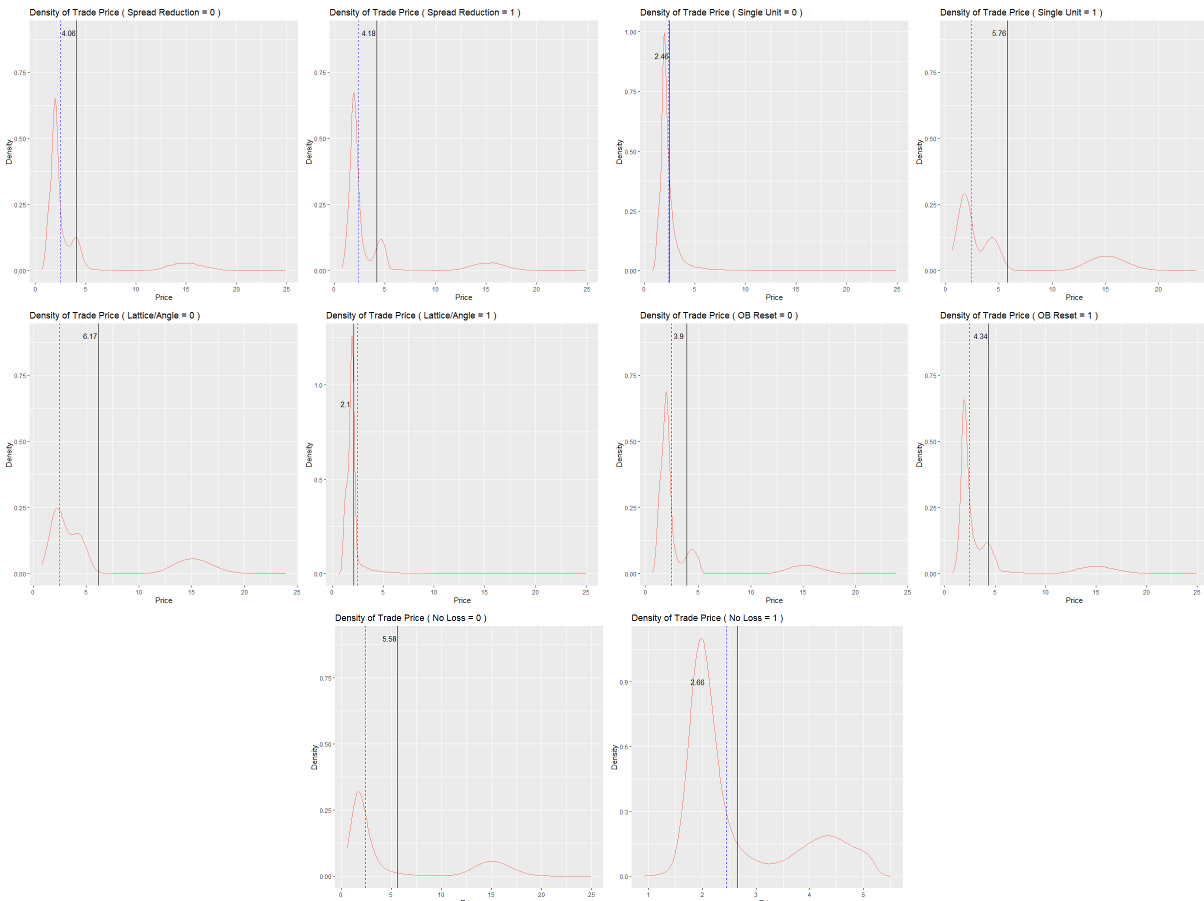

Figure E.1: Round-average price densities. Blue line is CE price and black line is subset average. Outliers with round-average price > 25 omitted (top 0.8%).

Appendix F Rule-type Pairings and Efficiency

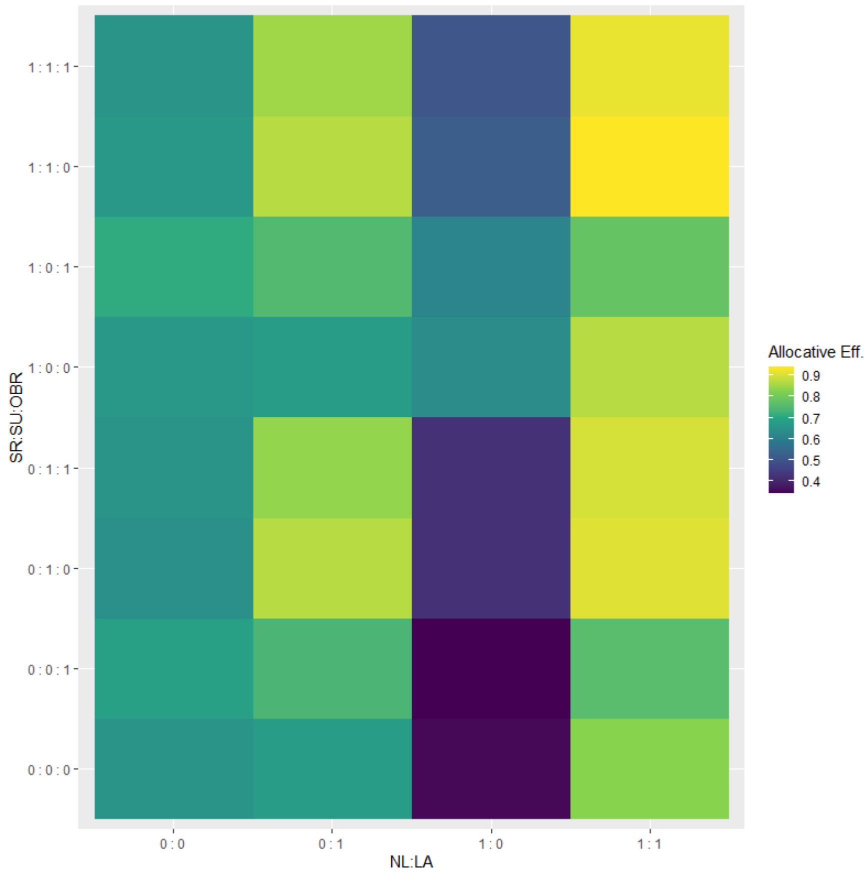

Figure F.1: A heatmap of allocative efficiencies plotted over the SR:SU:OBR×NL:LA pairings. The SR:SU:OBR axis marks denote the indicator values for each of the three market-oriented rules. Similarly, NL:LA axis marks show the indicator values for the behaviorally-oriented rules.

As alluded to in the primary analysis, the five rules defining this paper's simulation investigation can be partitioned into two categories: market-oriented rules and behaviorally-oriented rules. Section 5.2 suggests evidence of heterogeneous impacts on key price and efficiency measures between the two categories. Figure F.1 presents a heatmap of efficiencies across the 32 treatments, with one axis reporting the enforced market rules and the other axis showing the enforced behavioral rules. The figure serves as a re-imagination of the estimates presented in the allocative efficiency column of Table 3.

Clearly, column by column comparison reveals far more heterogeneous hues as compared to a row based adjustment. A one step move on the NL:LA axis reveals large consequences in equilibration, with within-row deviations over 0.3 in all but two rows (1:0:0 and 1:0:1). Row to row adjustments are far more tame, though two pairs of row clusters are apparent. It turns out these clusters are exactly partitioned by the inclusion of SU as an enforced rule. Within row, SU=0 rows show muted hue adjustment across columns[30], while SU=1 rows show adjustments of close to 0.5 when moving from the 1:0 to 1:1 column.

[30] SU=0 rows which also satisfy SR=0 exhibit a large difference in the 1:0 column, but are rather homogeneous in the other three columns.

Appendix G   Efficiency and Per-Unit Avg. Price Bivariate

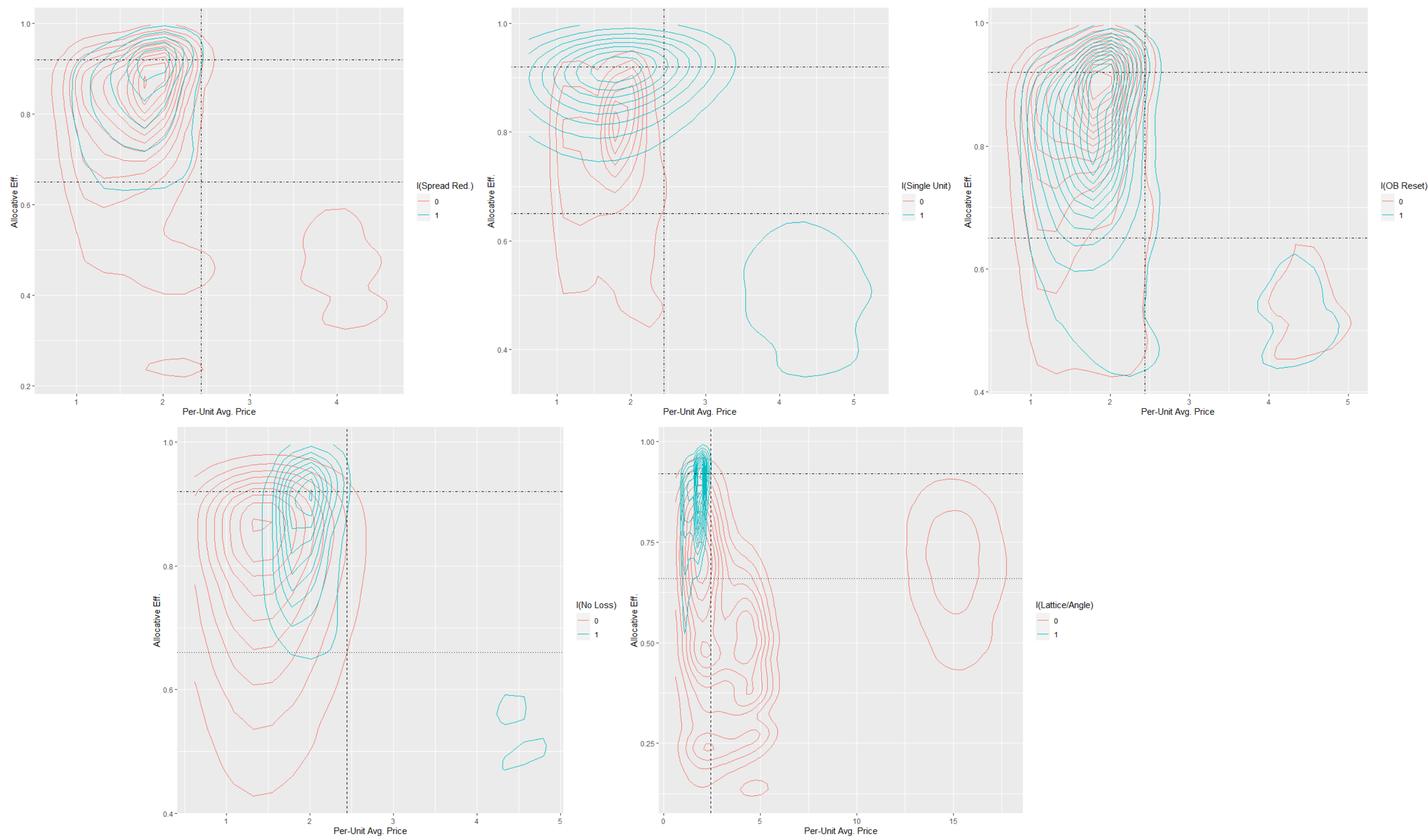

Figure G.1: Bivariate densities over average price and allocative efficiency. Densities are separated based on inclusion or exclusion of each rule. The upper horizontal dotted line shows the allocative efficiency of the 1:1:1:1:1 markets; the lower horizontal dotted line shows the allocative efficiency of the 0:0:0:0:0 markets; the vertical dotted line shows the competitive equilibrium price at endowment.

Appendix H Robustness Runs

H.1 1k-entries per round

Outcomes		*Rule Breakdowns*			
		None Enforced	Only Market	Only Behavioral	All Enforced
Prices.					
	Average Price	2.15 (0.31)	15.29 (0.96)	2.13 (0.10)	2.09 (0.14)
	Per-Unit Avg.	1.59 (0.21)	15.29 (0.96)	1.99 (0.12)	2.09 (0.14)
	$\lvert Price - CE \rvert$	1.35 (0.21)	12.93 (0.95)	0.51 (0.07)	0.59 (0.08)
	RMSE	1.78 (0.82)	15.15 (1.13)	0.64 (0.08)	0.77 (0.10)
Volume.					
	Order Size	15.08 (0.68)	1.00 (0.00)	5.69 (0.33)	1.00 (0.00)
	# Trades	59.56 (8.47)	86.15 (7.08)	51.90 (6.46)	27.93 (2.29)
	Trade Size	3.61 (0.48)	1.00 (0.00)	0.37 (0.05)	1.00 (0.00)
Efficiencies.					
	Allocative Eff.	0.67 (0.17)	0.66 (0.17)	0.95 (0.02)	0.99 (0.00)
	Distance Eff.	0.21 (0.14)	0.28 (0.11)	0.77 (0.03)	0.86 (0.03)
	Seller MRS	2.05 (0.47)	1.82 (0.26)	2.05 (0.07)	2.43 (0.08)
	Buyer MRS	3.07 (0.70)	3.47 (0.69)	2.93 (0.09)	2.52 (0.07)

Table H.1: Outcome averages by treatment with 1000 entries per market round.

Outcomes		*Rule Breakdowns*				
		Only NL	Only OBR	Only LA	Only SU	Only SR
Prices.						
	Average Price	2.41 (0.44)	3.27 (2.24)	1.65 (2.08)	15.08 (0.88)	1.94 (1.23)
	Per-Unit Avg.	2.34 (0.40)	1.7 (0.26)	1.11 (0.07)	15.08 (0.88)	1.56 (0.16)
	$\lvert Price - CE \rvert$	0.64 (0.26)	2.43 (2.20)	1.68 (2.08)	12.64 (0.88)	1.20 (1.23)
	RMSE	0.75 (0.29)	6.08 (13.76)	5.88 (43.79)	14.09 (0.99)	2.15 (10.76)
Volume.						
	Order Size	15.19 (0.51)	14.87 (0.68)	42.50 (209.04)	1.00 (0.00)	8.78 (0.90)
	# Trades	3.92 (1.17)	45.67 (6.12)	466.48 (16.79)	89.47 (8.39)	80.93 (8.88)
	Trade Size	3.61 (0.48)	1.00 (0.00)	0.37 (0.05)	1.00 (0.00)	
Efficiencies.						
	Allocative Eff.	0.65 (0.15)	0.71 (0.16)	0.69 (0.18)	0.65 (0.17)	0.67 (0.17)
	Distance Eff.	0.48 (0.13)	0.21 (0.14)	0.01 (0.18)	0.27 (0.10)	0.27 (0.14)
	Seller MRS	1.70 (0.30)	2.06 (0.45)	2.08 (0.53)	1.78 (0.24)	2.05 (0.44)
	Buyer MRS	3.52 (0.55)	3.03 (0.58)	3.08 (0.71)	3.62 (0.65)	3.06 (0.58)

Table H.2: Outcome averages by treatment with 1000 entries per market round.

H.2 GSS Parameters

Outcomes		*Rule Breakdowns*			
		None Enforced	Only Market	Only Behavioral	All Enforced
Prices.					
	Average Price	1.22 (0.36)	29.77 (3.49)	1.34 (0.17)	1.74 (0.29)
	Per-Unit Avg.	0.90 (0.20)	29.77 (3.49)	0.99 (0.14)	1.74 (0.29)
	$\|Price - CE\|$	0.83 (0.28)	28.33 (3.49)	0.56 (0.12)	1.16 (0.21)
	RMSE	1.06 (0.85)	32.13 (3.99)	0.77 (0.31)	1.82 (0.48)
Volume.					
	Order Size	53.52 (3.75)	1.00 (0.00)	15.28 (1.80)	1.00 (0.00)
	# Trades	18.38 (4.70)	25.65 (3.78)	38.21 (5.67)	44.54 (3.36)
	Trade Size	12.70 (2.65)	1.00 (0.00)	2.63 (0.46)	1.00 (0.00)
Efficiencies.					
	Allocative Eff.	0.67 (0.14)	0.59 (0.14)	0.95 (0.02)	0.76 (0.04)
	Distance Eff.	0.38 (0.11)	0.21 (0.06)	0.72 (0.06)	0.52 (0.04)
	Seller MRS	0.70 (0.37)	0.51 (0.12)	1.13 (0.14)	0.55 (0.06)
	Buyer MRS	4.53 (1.79)	7.43 (2.64)	2.65 (0.28)	4.97 (0.51)

Table H.3: Outcome averages by treatment with Gode et al. (2004) parameters.

H.3 Gjerstad (2013) Parametrizations

Outcomes		*Rule Breakdowns*			
		None Enforced	Only Market	Only Behavioral	All Enforced
Prices.					
	Average Price	136.68 (925.86)	1148.83 (142.17)	2.58 (1.09)	3.67 (3.45)
	Per-Unit Avg.	87.00 (18.09)	1148.83 (142.17)	1.05 (0.28)	3.67 (3.45)
	$\|Price - CE\|$	84.45 (925.35)	1058.99 (141.75)	88.49 (1.02)	88.49 (2.38)
	RMSE	224.92 (3698.81)	1284.4 (159.53)	3.23 (7.76)	10.78 (19.12)
Volume.					
	Order Size	20.08 (1.42)	1.00 (0.00)	1.23 (0.18)	1.00 (0.00)
	# Trades	19.44 (4.59)	23.53 (3.85)	72.74 (8.39)	54.45 (4.05)
	Trade Size	5.06 (1.01)	1.00 (0.00)	0.91 (0.14)	1.00 (0.00)
Efficiencies.					
	Allocative Eff.	0.77 (0.10)	0.83 (0.09)	0.89 (0.02)	0.88 (0.02)
	Distance Eff.	0.21 (0.16)	-0.04 (0.14)	-0.16 (0.04)	0.05 (0.07)
	Seller MRS	383.82 (596.5)	149.05 (55.97)	2.12 (0.68)	3.23 (5.31)
	Buyer MRS	131.5 (4058.98)	1243 (2850.98)	42.22 (43.07)	57.2 (8.76)

Table H.4: Outcome averages by treatment with Gjerstad (2013) preference functional form and parametrization; the CE price is 91.

Outcomes		*Rule Breakdowns*			
		None Enforced	Only Market	Only Behavioral	All Enforced
Prices.					
	Average Price	1.19 (0.42)	11.39 (1.40)	1.03 (1.95)	1.54 (0.37)
	Per-Unit Avg.	0.87 (0.18)	11.39 (1.40)	0.68 (0.17)	1.54 (0.37)
	$\lvert Price - CE \rvert$	0.67 (0.39)	10.50 (1.39)	0.52 (1.95)	0.90 (0.31)
	RMSE	1.02 (1.49)	11.97 (1.60)	1.43 (11.67)	1.71 (0.76)
Volume.					
	Order Size	20.03 (1.41)	1.00 (0.00)	4.23 (0.57)	1.00 (0.00)
	# Trades	19.46 (4.56)	23.44 (3.82)	38.50 (6.53)	29.02 (3.06)
	Trade Size	5.05 (0.99)	1.00 (0.00)	1.16 (0.30)	1.00 (0.00)
Efficiencies.					
	Allocative Eff.	0.77 (0.09)	0.83(0.09)	0.99 (0.02)	1.00 (0.00)
	Distance Eff.	0.21 (0.16)	-0.04 (0.14)	0.72 (0.11)	0.65 (0.14)
	Seller MRS	3.70 (5.25)	1.49 (0.54)	0.78 (0.18)	1.24 (0.20)
	Buyer MRS	0.59 (1.66)	12.64 (38.4)	0.84 (0.36)	1.08 (0.19)

Table H.5: Outcome averages by treatment with Gjerstad (2013) parameters normalized to reduce the numeraire supply and CE price by a factor of 100. Y endowments were reduced to be 18 per buyer/seller pair and the CE price is 0.91 as opposed to 91.